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Journal of Advances in Applied Computational Mechanics and Engineering

Aims and Scope

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Contents

New Coefficient Inequalities for Certain Subclasses of p-Valent Analytic Functions

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Abstract:

The object of the present paper is to derive new coefficient inequalities for certain subclasses of p valent functions defined in the open unit disk U. Our results are generalized of the previous theorems given by J. Clunie and F.R. Keogh [1], by H. Silverman [3] and by M. Nunokawa et al. [2].

Keywords: Analytic functions, p valently starlike of order , p valently convex of order , coefficient inequalities.

1. INTRODUCTION

Let Ap denote the class of the form $f(z) = \sum_{n=p}^{\infty} a_n z^n, \quad \left(a_p = 1,\ p,\ n \in \mathbb{N} = \{1,2,...\}\right),$ (1.1)

which are analytic and p- valent in the open disk $\mathcal{U} = \left\{z \in \mathbb{C} : |z| < 1\right\}.$ We note that

A1=A. A function f e Ap is said to be p - valently starlike of order $\alpha\left(0 \le \alpha < p\right)$

if and only if $\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, \quad (z \in \mathcal{U}).$ The class of all such functions are denote by

Here $\mathcal{S}_1^*\left(\alpha\right) = \mathcal{S}^*\left(\alpha\right)$ and $\mathcal{S}^*\left(0\right) = \mathcal{S}^*$ are the classes of starlike function of o: $\mathcal{S}_p^*(\alpha)$.

$\alpha\left(0 \le \alpha < 1\right)$ and starlike function, respectively. On the other hand, a function f e Ap is said

to be p- valently convex of order $\alpha\left(0 \le \alpha < p\right)$ if and only if

$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha, \quad (z \in \mathcal{U}).$ Let Cp (α) denote the class of all those functions.

Also C1 (α)=C( ) and C (0) = C are the classes of convex function of order $\alpha\left(0 \le \alpha < 1\right)$

and convex function, respectively. Clunie and Keogh [1] (also Silverman [3]) have the following results: If f(z) e A satisfies $\sum_{n=2}^{\infty} n\left|a_n\right| \leq 1,$ then f(z) is univalent and starlike in U. If f(z) A satisfies $\sum_{n=2}^{\infty} n^2\left|a_n\right| \leq 1,$ then f(z) is univalent and convex in U. Nunokawa et al. [2] have proved the following results: Let f(z) be of the class A and $\max_{n\geq 1}\left|a_n\right| = t\left|a_t\right|$. If $f(z) \in$ A satisfies $\sum_{n=1,\ n\neq t}^{\infty} \left(\left|n-t\right|+t\right)\left|a_n\right| \leq t\left|a_t\right|,$ then f(z) is univalent and starlike in U. Let f(z) be of the class A and $\max_{n\geq 1} n^2\left|a_n\right| = t^2\left|a_t\right|$. If $f(z) \in \mathcal{A}$ satisfies $\sum_{n=1,\ n\neq t}^{\infty} n\left(\left|n-t\right|+t\right)\left|a_n\right| \leq t^2\left|a_t\right|,$

then f(z) is univalent and convex in U. In the present investigation, we consider new coefficient inequalities for functions f(z) to be p valently starlike of order and p valently convex of order in U.

2. COEFFICIENT INEQUALITIES

Our first result for functions f(z) to be p valently starlike of order in U is contained in the following Theorem 2.1.

Theorem 2.1. Let f(z) be in the class Ap and $\max_{n\geq p} n\left|a_n\right| = \left(t+p-1\right)\left|a_{t+p-1}\right|.$

If f(z) Ap satisfies the following inequality

$$\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} \left(\left|n-t-p+1\right|+t+p-1+\alpha\right)\left|a_n\right| \leq \left(t-p+1+\alpha\right)\left|a_{t+p-1}\right|,$$

then f(z) is p valently starlike of order in U.

Proof: Applying the maximum principle of analytic the following inequality is hold on |z|=1

$$\left|zf'(z)-tf(z)-\left(p-1\right)f(z)\right|-\left|tf(z)\right|-\left|\left(p-1\right)f(z)\right|+\left|\alpha f(z)\right|$$

$$=\left|\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} \left(n-t-p+1\right)a_n z^n\right|-t\left|\sum_{n=p}^{\infty} a_n z^n\right|-\left(p-1\right)\left|\sum_{n=p}^{\infty} a_n z^n\right|+\alpha\left|\sum_{n=p}^{\infty} a_n z^n\right|$$

$$\leq \sum_{\substack{n=p, \\ n\neq t+p-1}}^{\infty} |n-t-p+1||a_n||z^n| - t\left(|a_{t+p-1}||z|^{t+p-1} - \sum_{\substack{n=p, \\ n\neq t+p-1}}^{\infty} |a_n||z^n|\right)$$

$$-(p-1)\left(|a_{t+p-1}||z|^{t+p-1} - \sum_{\substack{n=p, \\ n\neq t+p-1}}^{\infty} |a_n||z^n|\right) + \alpha\left(|a_{t+p-1}||z|^{t+p-1} - \sum_{\substack{n=p, \\ n\neq t+p-1}}^{\infty} |a_n||z^n|\right)$$

$$= \sum_{\substack{n=p, \\ n\neq t+p-1}}^{\infty} \left(|n-t-p+1| + t + p - 1 + \alpha\right)|a_n| - \left(t-p+1+\alpha\right)|a_{t+p-1}| \leq 0.$$

Therefore, it follows that the following inequality $\left|\frac{zf'(z)}{f(z)} - t - (p-1)\right| \leq t + (p-1) - \alpha$

holds for all z U. This shows that f(z) is p valently starlike of order in U.

If we take $\alpha = 0$ in the Theorem 2.1., we get the following corollary. **Corollary 2.2.** Let f(z) be in the

class Ap and $\max_{n\geq p} n|a_n| = (t+p-1)|a_{t+p-1}|$. If f(z) Ap satisfies the following inequality

$\sum_{\substack{n=p, \\ n\neq t+p-1}}^{\infty} \left(|n-t-p+1| + t + p - 1\right)|a_n| \leq (t-p+1)|a_{t+p-1}|$, then f(z) is p valently starlike in U.

For p=1 in the Theorem 2.1., we have the following corollary. **Corollary 2.3.** Let f(z) be in the class

A and $\max_{n\geq 1} n|a_n| = t|a_t|$. If f(z) e A satisfies the following inequality

$\sum_{n=1, n\neq t}^{\infty} \left(|n-t| + t + \alpha\right)|a_n| \leq (t+\alpha)|a_t|$, then f(z) is starlike of order in U.

Next, we derive the coefficient condition for functions f(z) to be p valently convex of order in U is contained in the Theorem 2.4 as given below. **Theorem 2.4.** Let f(z) be in the class Ap and

$\max_{n\geq p} n^2\left|a_n\right| = \left(t+p-1\right)^2\left|a_{t+p-1}\right|$. If $f(z) \in \mathcal{A}_p$ satisfies the following inequality

$$\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left(\left|n-t-p+1\right|+t+p-1+\alpha\right)\left|a_n\right| \leq \left(t+p-1\right)\left(t+p-1-\alpha\right)\left|a_{t+p-1}\right|, \quad (2.2)$$

then $f(z)$ is $p-$ valently convex of order α in $\mathcal{U}$.

Proof: Applying the maximum principle of analytic functions, the following inequality is hold on $|z|=1$

$$\left|zf''(z)+f'(z)-tf'(z)-\left(p-1\right)f'(z)\right|-\left|tf'(z)\right|-\left|\left(p-1\right)f'(z)\right|+\left|\alpha f'(z)\right|$$

$$=\left|\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty}\left[n\left(n-t-p+1\right)\right]a_n z^{n-1}\right|-t\left|\sum_{n=p}^{\infty} n a_n z^{n-1}\right|$$

$$-\left(p-1\right)\left|\sum_{n=p}^{\infty} n a_n z^{n-1}\right|+\alpha\left|\sum_{n=p}^{\infty} n a_n z^{n-1}\right|$$

$$\leq \sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left|n-t-p+1\right|\left|a_n\right|\left|z^{n-1}\right|-t$$

$$\left(\left(t+p-1\right)\left|a_{t+p-1}\right|\left|z\right|^{t+p-1}-\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left|a_n\right|\left|z^{n-1}\right|\right)$$

$$-\left(p-1\right)\left(\left(t+p-1\right)\left|a_{t+p-1}\right|\left|z\right|^{t+p-1}-\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left|a_n\right|\left|z^{n-1}\right|\right)$$

$$+\alpha\left(\left(t+p-1\right)\left|a_{t+p-1}\right|\left|z\right|^{t+p-1}-\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left|a_n\right|\left|z^{n-1}\right|\right)$$

$$= \sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left(\left|n-t-p+1\right|+t+p-1+\alpha\right)\left|a_n\right|$$

$$-\left(t+p-1\right)\left(t+p-1-\alpha\right)\left|a_{t+p-1}\right| \leq 0.$$

Therefore, it follows that the follwing inequality

$$\left|\left(1+\frac{zf''(z)}{f'(z)}\right)-t-\left(p-1\right)\right| \leq t+\left(p-1\right)-\alpha$$

holds for all $z \in \mathcal{U}$. This shows that $f(z)$ is $p-$ valently convex of order α in $\mathcal{U}$.

By taking =0 in the Theorem 2.4, we get the following corollary. Corollary 2.5. Let f(z) be in the class

Ap and $\max_{n\geq p} n^2\left|a_n\right| = \left(t+p-1\right)^2\left|a_{t+p-1}\right|.$

NEW COEFFICIENT INEQUALITIES

If $f(z) \in \mathcal{A}_p$ satisfies the following inequality

$\sum_{\substack{n=p,\\ n\neq t+p-1}}^{\infty} n\left(\left|n-t-p+1\right|+t+p-1\right)\left|a_n\right| \leq \left(t+p-1\right)^2\left|a_{t+p-1}\right|,$ then $f(z)$ is $p-$ valently

convex in U. By taking p=1 in the Theorem 2.4, we get the following corollary.

Corollary 2.6. Let f(z) be in the class A and

$$\max_{n\geq 1} n^2\left|a_n\right| = t^2\left|a_t\right|.$$

If $f(z) \in \mathcal{A}$ satisfies the following inequality

$$\sum_{n=1, n \neq t}^{\infty} n\left(\left|n-t\right|+t+\alpha\right)\left|a_n\right| \leq t\left(t-\alpha\right)\left|a_t\right|,$$ then f(z) is convex of order in U.

Remark 2.7. By considering some special values for the parameters , p and t, we can deduce the following results.

In the Theorem 2.1. and Theorem 2.4., for p=1 and α =0, we get the result given by Nunokawa et al. [2].

In the Theorem 2.1. and Theorem 2.4., for p=1, =0 and t=1, we obtain the result given by Clunie and Keogh [1] (also Silverman [3]).

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The Methodological Aspect of Development and Application Multivariate Classification G-Mode for Analyses Geochemical Trend

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Abstract:

This article is unique G-mode of multidimensional classification method and its application in the analysis of hydrogeochemistry Donetsk basin. He has the following main advantages over other methods of classification: does not require a priori information for classification of observations; earmarks homogeneous observing classes and subclasses; evaluates information weight of each indicator, determines the distance between the homogeneous taxon's; assessment of the descriptiveness of the sign of the classification and others. G-method is widely used to analyze geochemical, environmental, kosmochemical, distance and other types of information. The method successfully used in examining objects, phenomena and processes on Earth, Moon, Mars, Saturn, comets, asteroids and deep space. The results of the use of G-method in analysis of hydrogeological data for the Donetsk basin identified direct and inverse geochemical zonality. This shows that in the region can be discovered oil and gas accumulations.

Keywords: Methodology, geochemical investigation, modeling, classification of G-mode, Donbass, hydrogeochemical zonality.

Quantitative nature of modern geochemical information creates opportunities to use mathematical techniques and computer technology for processing of raw data and justification of conclusions about the patterns of distribution of chemical elements in rocks and underground water. However, there are serious difficulties due to the complexity of the matters dealt with in geochemistry (in geology) objects and processes, the shortcomings of the existing classifications, disadvantages of many geological concepts, etc.

MATHEMATICAL MODELING

Application of mathematics in geochemistry, according to most researchers, should be considered primarily as an application of the method of mathematical modeling. This means that it is necessary to establish similar object (process) and the model. The model is designed for the study of the object and should be similar to the signs of that object (the other properties-featured model is different from the otherwise it ceases to be just a model).

Under the mathematical modeling in geochemistry is understood this way of discovering the laws of space-time distribution and migration of chemical elements in natural objects, according to certain rules when a mathematical description of some of the geochemical properties of the object or process, and further: 1) on the basis of a study of this description are improved and expanded (often projected) geochemical knowledge on the same object or process; 2) on the basis of a similarity of the new object known object on mathematical description of geochemical properties, prediction of other properties for the new object [2.6].

In each case, can be used by different mathematical models, but most commonly in the geochemistry of applied probability, as the concentration of a chemical element geological area at this point is influenced by many factors, which are practically impossible to deterministically. Probabilistic-statistical model consists of the following main parts: 1) deterministic part that describes the changes in the chemical composition of natural objects and processes under the influence of the leading factors; 2) random part of the describing the actual random changes in the chemical composition by secondary factors; 3) random part, showing the observed changes in composition due to errors.

From the point of view of the geochemistry is the most interesting study of the first two parts of the model (deterministic and random), and the third part due to errors, distorts the actual distribution of the concentrations of chemical elements and prevents the researcher. Under this model, the concentration of chemical elements in some geological space can be written as follows:

C (x, y, z, t) =(μ)(x, y, z, t) +(v)(x, y, z, t) + V (x, y, z, t), where C (x, y, z, t) is the chemical element observed at some point geological space

with coordinates x, y, z, and the coordinate time t;(μ)(x, y, z, t) is the concentration of the element at coordinates x, y, z, t, due to major natural factors;(v)(x, y, z, t) is a real element of concentration deviation(μ)due to occasional minor factors; (x, y, z, t)-deviation due to errors by the geological space with coordinates x, y, z, t, we get more as compact expression:

$$(C)_{(i)} = (\mu)_{(i)} + (\nu)_i + \upsilon_{(i)} .$$

Specialist seeks within the studied object, set the first legitimate change of (μ) , for example, uses a trend-analysis, moving average, factor analysis, etc. the most successfully used classification procedures with homogenous geological parts of space, i.e. the selection of parts with permanent(μ)Now the model can be written as follows:

$$Ci = \mu + Vi + (vi)$$

Thus, geologist, homogeneous parts of an object gets a reasonable statistical method of their geochemical characteristics and objective comparison of homogeneous parts-natural ways to identify changes for the object.

Statistical analysis of geochemical data to draw conclusions of the specified reliability. It includes the following main stages: planning, data processing management of observation, evaluation and quality control of primary geochemical information, verification of the homogeneity of the sample selection models (laws) distribution of components in natural objects, estimate the parameters of distributions, distributions, comparison study of relationships between content components, application of multidimensional data processing techniques (various modifications of the classification methods, factorial analysis, allocation of homogeneous statistical aggregates, pattern recognition, etc.).

G–MODE MULTIDIMENSIONAL CLASSIFICATION METHOD OF OBSERVATION

When studying rocks, minerals, water, and other natural objects and processes one of the most important is the task of building their classifications that can be kept to determine the taxonomic structure of the geological area. For selection in the space-time coordinates of similar taxa in geology multidimensional mathematical methods. These methods are based on a variety of statistical and heuristic principles, each of which has specific advantages and disadvantages.

Multidimensional classification method of geological observations [1.2], the following general requirements are to ensure that:

- construction of classification in the absence of a priori information about the taxonomic structure of the observations;
- the use of dependent indicators and observations;
- allocation of taxonomic structures of various levels;
- unlimited ratio the number of indicators (M) and the number of observations (N);
- use of the information on changing average values, and the relationship between signs;
- assessment of the descriptiveness of the sign of the classification;
- evaluation of similarity-difference between homogeneous taxonomy;
- classification of new observations about the taxonomic structure.

A detailed analysis of the different classification methods and their problems can be found in the literature, here we will focus on the new (G)multidimensional classification method, developed by work [1, 2, 6, 7].

The basis for the classification procedure, called G method based on the criterion of Gavrishin - Z2 with distribution of quasi- 2 [1]:

$$Z^2 = \frac{M}{\sum_{SK} r_{SK}^2} \sum_{ij} Z_{ij}^2 = K \sum_{ij} Z_{ij}^2,$$

$$Z_{ij} = \frac{x_{ij} - \mu_i}{\sigma_i},$$

where xij -value indicator I (1.2, ..., M), j (1.2, ..., N); i and (i) -average value and standard deviation (i) in a homogeneous classroom observations; rSK -correlation coefficients between indicator s and k.

The hypothesis of one or more observations to this homogeneous class is rejected, if the computed

$Z^2 > \chi^2_{q,f}$, where $\chi^2_{q,f}$ - critical distribution X2 , when the significance level q and the number of

degrees of freedom (f):

$$f = N . M . K .$$

To simplify the procedures for the use of the distribution X2 his conversion was applied in normal

parameters (0.1): $G = \sqrt{2z^2} - \sqrt{2f-1}.$

If (G)>Gq the surveillance data do not belong to the homogeneous taxon, where Gq -critical values for level of significance q.

Classification procedure (G-mode) is reduced to the following main operations: selecting a coordinate system in which the transformation of the multidimensional space of attributive to the distribution of Z2, and find the center of the first homogeneous taxon; the transformation of the coordinate system and finding all the observations of the first homogeneous taxon; repeat these operations for the observations, which were not included in previous similar taxa; evaluation of similarity-difference between homogeneous taxonomy in each and all indications simultaneously; estimation of informatively of the taxonomic structure; repeat all steps for the different levels of reliability allocation of similar taxa.

Largest G all observations found that belong to this homogeneous taxon. Modifying the critical radius of a homogeneous taxon (Gq) you can get different levels of detail and classification of varying degrees of homogeneity of the sub-family G lower values. Gq the higher the homogeneity of the taxa, more detail of classification, but a lower reliability validity of differences between taxonomy.

Evaluation of similarity-difference between homogeneous taxonomy is also based on criteria Z2 and is determining the international taxonomic variance for each account and all the indicators together. Media weight characteristics can be estimated as the sum of the difference between the uniform taxonomy [2].

Largest Z2 or G can be evaluated any number of new observations belonging to the fusion of units, that is, to produce a classification of observations. G- method is implemented in the form of computer technology AGAT to automatically build a multidimensional classification of observations at different levels of detail, and successfully apply to natural and natural-human systems on Earth, Moon, Mars, asteroids, and comets in deep space for astrophysical, kosmochemical, remote, hydrogeochemical, geological and other data [1, 2, 6, 7]. This paper considers the technique and results of application G-method based on the analysis of hydrogeochemistry of Donbass.

GEOCHEMICAL ZONING OF GROUNDWATER

The most important tributaries in the coal mines of Donetsk basin form the water of Carboniferous, Cretaceous, Paleogene and Neogene deposits. To identify and quantify the vertical geochemical zonality groundwater systems listed have been used chemical analysis of samples of water taken from wells [4]. The application of a consistent classification modeling using G-method identified two main trends in the chemical composition of the groundwater, which are similar to water the aquifer complexes; below I'll elaborate on the geochemical zonation of water in Carboniferous deposits ©. Since changing contents of many of the features in depth is curved, the specifications of the subject was offered the following function, which allows you to describe the positive, negative, positive in the negative (and vice versa), and the periodic function:

$$Y = \sum_{i} a_i \cdot 10^{-\frac{(\lg x - b_i)^2}{c_i}},$$

where a(I) - factors characterizing the modal (top) the value of the function; b(I) -the coefficients describing the location of modal values on the x axis; c(I) - factors characterizing the slope of the regression line; inumber of the modal value.

Water Carboniferous deposits on the territory are very diverse chemical composition: water varies from hydrocarbonate calcium to chloride sodium, salinity from 0.2 to 57.2 g/l, Cl - -0.012-35.6, Na+ - 0,002-17.6 g/l, etc. (Table 1). Distribution of the contents of components does not correspond to the normal model and correlation may be curvilinear. For example, for So4 2- the transition is clearly positive due to the depth (H) of mineralization and (M) the negative. A very strong correlation with salinity (r > 0.95) discover Na+, Cl-, Ca2+, Mg2+, with a depth of strongly connected M, Cl– and Na+. However, you can see that at significant depths (more than 400-500 m) and salinity (M > 50 g/l), and fresh water (M-2-3 g/l).

Table 1: Groundwater Chemical Composition of Carboniferous Deposits

Component	X_m	Me	X_{min}	X_{max}	S
pH	7.7	7.7	6.4.	8.6	0.5
HCO_3^-	358	352	77	947	135
SO_4^{2-}	485	400	15	1427	405
Cl^-	2366	243	12	35636	7086
Ca^{2+}	337	153	20	4084	716
Mg^{2+}	140	82	4	11145	227
Na^+	1237	252	2	17582	3316
(M)	4729	1611	178	57419	11149
(H)	124	75.5	3	922	163

Note: in tables X_m-arithmetic means, Me - median, x_{min} and x_{max} -the minimum and maximum value, S-standard deviation (in mg/l, H-depth in meters).

With the help of G-method of computer technology AGAT in the waters of Carboniferous deposits by chemical composition has been allocated 11 similar geochemical types plus 1 anomalous (Table 2).

Interesting patterns identified in examining detailed water classification by type in the coordinates of "the depth (H)-content component-mineralization (M)". Surely there are two main trends in geochemical

Table 2: Geochemical Composition of Carboniferous Deposits of Groundwater (mg/l and %-mole)

Tendention	View	H	Ph	Components						(M)
				HCO_3^-	SO_4^{2-}	Cl^-	Ca^{2+}	Mg^{2+}	Na^+	
1	1.6	40	7.1.	404	355	46	111	47	134	900
				44	48	8	37	25	38	
	1.2	79	7.7	351	425	213	158	58	183	1200
				28	43	29	38	24	38	
	1.3	82	7.7	395	790	315	185	99	333	1900
				20	52	28	29	26	45	
	2.2	102	8.0	380	810	1180	275	168	660	3300
				11	30	59	24	25	51	
	2.1	106	8.0	387	1160	786	316	177	506	3010
				12	46	42	30	28	42	
	3.1	333	8.3	293	86	14700	1300	532	6920	24000
				1	0.4	98.6	18	11	71	
	A. 1	730	8.4	290	73	33930	3500	1100	16000	55000
				0.5	0.2	99.3	18	10	72	
2	1.4	28	6.8	247	65	18	71	16	25	320
				68	24	8	59	22	19	
	1.1	50	7.4	382	114	57	65	41	82	550
				61	23	16	32	33	35	
	1.5	73	7.2	310	176	56	90	34	68	580
				49	36	15	44	28	28	
	1.7	100	7.6	372	450	173	98	118	128	1153
				30	46	24	24	48	28	
	4.1	380	7.8	548	418	1250	46	38	1100	3100
				17	16	67	4	6	90	

composition of groundwater of Carboniferous deposits by depth [3], which reflects the direct and inverse vertical geochemical zonality groundwater. In Table 2 geochemical types are positioned as the depth of water and geochemical trends.

The first trend is a typical representative of direct vertical geochemical zonality and, despite the high heterogeneity of the composition of water, it is characterized by a natural transition from low mineralized hydrocarbonate and hydrocarbonatesulphate mixed cation composition of waters to chloride-sulphate and further mineralized chloride sodium **(Table 2)** with increasing depth

A central role in the formation of the chemical composition of the waters of Carboniferous deposits on the first trends are Cl– and Na+, the contents of which increases with depth, grows so does the amount of water pH and decreases So24 . The coefficients are reduced dramatically-relationships rHCO3-/rCa2+ +rMg2+ from 0.8 to 0.02 and rSO4 2-/rCl- from 6 up to 0.002. Clearly the patterns changes in the composition of the waters with depth are visible on the equations (Table 3). Increase of SO4 2- turns out (Hmax = 90 m), the pair correlation coefficient of no significant, while curvilinear correlation factor (r1 = 0.66) and the regression equation is as follows (Table

$$SO_4 = 790 \cdot 10^{-\frac{(\lg H - 1{,}96)^2}{0.6}}.$$

On the genesis of the first geochemical trends, you can quite confidently say that from a depth of 150200m starting to wane of infiltration in the formation of the chemical composition of groundwater and the increasing role of the sedimentation. This has an impact on reducing the contents in the waters So4 2- and the HCO3 2- and increasing Cl- and Na+; water of II type by O.A. Alekin go III contents; of the J typically 510 mg/l, Br -20-30 mg/l. In the open part of the Eastern Donbass to mineralized water chloride sodium occurs at depths of about 1 km in the outlying parts of the basin depth of mineralized waters is much closer to the surface.

The second trend reflects a backward vertical geochemical zonality of the underground water of Carboniferous deposits, where a slight increase in salinity with depth gives way to its reduction and the formation water soda type. These trends are described well curved exponential function (Table 3) with high correlation coefficients (ri) parametric equations. The regression equation editing, the following patterns are clearly visible: the maximum mineralization is achieved at depths of 250-300 m; the HCO3- increases with depth and a maximum is reached where reliably predict fails; the content of So4 2- and Cl- max at depths of 200350 m and deeper decline; the content of Ca2+ and

Table 3: The Parameters of the Regression Curved Equations of Components Contents in Groundwater Carboniferous Deposits of Donbass on Depth

Trend	Component	$(a)_{(i)}$	$(b)_{(i)}$	$c_{(i)}$	r_i	$(H)_{max}$
1	HCO_3^-	372	2.06	8.23	0.3	115
	SO_4^{2-}	790	1.96	0.6	0.66	90
	Cl^-	32200	3.05	0.89	0.90	>1000
	Ca^{2+}	4550	3.5	1.97	0.82	>3000
	Mg^{2+}	1280	3.5	2.6	0.82	>3000
	Na^+	15950	3.1	1.0	0.92	>1000
	(M)	54000	3.13	1.16	0.90	>1000
2	HCO_3^-	525	3	8.2	0.46	>1000
	SO_4^{2-}	491	2.32	0.63	0.52	210
	Cl^-	1400	2.38	0.3	0.91	240
	Ca^{2+}	83	1.44	5.0	0.34	30
	Mg^{2+}	75	2.14.	0.7	0.62	140
	Na^+	1150	2.55	0.3	0.93	350
	(M)	3300	2.45	0.45	0.89	280

Note: $(a)_{(i)}$ – X maximum in mg/l, $(b)_{(i)}$ -the location of the maximum depth H (lgH) , c_i -the slope of the regression line, r_i - the coefficient of correlation curvilinear, $(H)_{max}$ -maximum depth in m.

Mg2+ is observed at depths of 100-250 m and decreased with depth.

The second type of vertical geochemical zonality of the water depth moving away from hydrocarbonate calcium to a sulphate hydrocarbonate and hydrocarbonate sulphate mixed cation composition and chloride sodium (soda) with a salinity of 2-3 g/l; a second type of water gives way to the first with high content of HCO3- and very low - Ca2+ and Mg2+ . The attitude now rHCO3-/rCa2+ +rMg2+ increases to 1.7, and attitude rSO4 2-/rCl- reduced only to 0.2 for the extrapolation of equations (Table 3) suggests that at depths of more than 1 km are hydrocarbonate sodium waters with a salinity of less than 1 g/l.

On the education of hydrogeochemical inversions and the emergence of the mineralized waters depths there are a variety of hypotheses, of which the most popular are: infiltration, juvenile, degidratation and evaporation-condensation. There are currently a large number of photographs clearly show the presence of a not much mineralized water depths associated with oil and gas deposits [5], which refer to the soda type and which are very close to the chemical appearance to the water as described above. The author believe that soda fresh water second geochemical trends in Donbass is most likely associated with the processes of condensation of water vapour from the gas phase

These waters are found at different depths in the Carboniferous, Cretaceous, Paleogene and Neogene sediments and often confined to zones of vertical tectonic fracturing. Now, taking the hypothesis of evaporation-condensation of the genesis soda water with a high content of HCO3- and very low - Ca2+ and Mg2+ you must acknowledge within Donetsk basin in certain traps oil and gas accumulations, as is the adjacent geological structures (Dnieper-Donetsk, Donetsk-Don, the Azov-Kuban basin, etc.).

Vertical geochemical zonality regularities of groundwater had a significant impact on the formation of mine water, forming a different direction.

Thus, this article is multidimensional classification a unique(G-mode)method and its application in the analysis of hydrogeochemistry Donetsk basin classification methods are the primary means of knowledge of the world around us. To know means to categorize.

Today, the constantly increasing number of studied objects, processes and phenomena, which are characterized by plenty of signs (chemical, physical, geological, biological,
environmental,
medical,
sociological, economical etc.). We have developed a unique G-classification method of multivariate observations, which lets you describe the new spatial and temporal patterns of forming objects, phenomena and processes, make discoveries, to formulate new laws, describing the genesis of, establish the diagnosis. It has the following main advantages over other methods of classification:

- construction of classification in the absence of a priori information about the taxonomic structure of the observations;
- the use of dependent traits and observations;
- allocation of taxonomic structures of various levels;
- unlimited ratio characteristics (M) and the number of observations (N);
- use of the information on changing average values, and the relationship between signs;
- score information of individual traits in the classification;
- evaluation of similarity-difference between homogeneous taxonomy’s;
- classification of new observations about the taxonomic structure.

(G-mode)-the method is widely used to analyze geochemical, environmental, kosmohemical, distance and other types of information. the method successfully used in examining objects, phenomena and processes on Earth, Moon, Mars, Saturn, comets, asteroids and deep space [6-8]. It can be successfully applied in studies in geology, geochemistry, geography, economics, biology, physics, chemistry and other fields of knowledge. The results of the use of G-method in the analysis of hydrogeological data for the Donetsk basin identified direct and inverse geochemical zonality. This shows that in the region can be discovered oil and gas accumulations.

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Uni-Type Modal Operators On Intuitionistic Fuzzy Sets

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Abstract:

Intuitionistic Fuzzy Modal Operator was defined by Atanassov, he introduced the generalization of these modal operators. After this study, some authors defined some modal operators which are called one type and two type modal operator on intuitionistic fuzzy sets. In these studies, some extensions and characteristic properties were obtained. In this paper we defined new operators and examine some properties of them. In view of conclusions, it is shown that these operators are both one type and two type modal operators on Intuitionistic Fuzzy Sets. So, these common type modal operators are called uni-type modal operators on Intuitionistic Fuzzy Sets.

Keywords: Diagram of modal operators, Intuitionistic fuzzy operators, uni-type modal operators.

1. INTRODUCTION

The theory of fuzzy sets (FSs) was first stated by Zadeh, [12], in 1965. Let X be a set then the function] 1 , 0 [X : μ is called a fuzzy set over X and it is shown by). X (FS μ For X x ,) x (μ is called the membership degree of x on A and the nonmembership degree is).

Atanassov [1] defined intuitionistic fuzzy sets (IFS) in 1983. While the nonmembership degree for each element of the universe is fixed in fuzzy set theory. In intuitionistic fuzzy set theory, nonmembership degree is a more or less independent degree; satisfying the condition that it is smaller than 1-membership degree. So, if X is a universe then there exist membersip and nonmembership degrees for each x X, respectively μ(x) and (x) such that 0 μ(x)+ (x) 1.

IFS A is determined with the membership and non membership of μA(x) FS(X), A(x) FS(X) for x X, resp. Although the sum of the degrees of membership and not being a member of an element in FS theory is 1. But, in IFS theory, this sum is less than 1. Besides this, if A IFS(X) then μ, FS(X) and 1 μ and 1 μ.

An IFS A is said to be contained in an IFS B notation A B) if and only if for all x X:μA(x) μB(x) and A(x) B(x). It is clear that A=B if and only if A B and B A.The intersection and the union of two IFSs A and B on X is defined as following;

$$A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: \ x \in X\}$$

$$A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: \ x \in X\}$$

Definition 1.1. [2] Let A IFS and A={<x,μA(x), A(x)>: x X}. The set

$$A^c = \{< x, \nu_A(x), \mu_A(x) >: \ x \in X\}$$

is called the complement of A.

The notion of Modal Operators on IFSs was firstly introduced by Atanassov [2].

Definition 1.2. [2] Let X be a set and A={<x,μA (x), A (x)>: x X} IFS(X).

$$1) \ \Box \ A = \{< x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2} >: \ x \in X\}$$

$$2) \ \Diamond \ A = \{< x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2} >: \ x \in X\}$$

After this definition, in 2001, Atanassov, in [3], defined the extension of these operators as following,

Definition 1.3. [3] Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: \ x \in X\} \in IFS(X), \ \alpha \in [0,1]$.

$$1) \ \lceil_\alpha A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha >: \ x \in X\}$$

$$2) \ \lfloor_\alpha A = \{< x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x) >: \ x \in X\}$$

In these operators and If we choose =1 2 , we get the operators and , resp. Therefore, the operators and are the extensions of the operators and resp. Some relationships between these operators were studied by several authors [9, 11] In 2004, the second extension of these operators was introduced by Dencheva in [9].

Definition 1.4. [9] Let X be a set, A={<x,μA (x), A (x)>: x X} IFS(X) and $\alpha,\beta\in$, [0,1]. The sets

$\lceil_{\alpha,\beta} A$ and $\lfloor_{\alpha,\beta} A$ are defined as follows:

1) $\lceil_{\alpha,\beta} A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + \beta >: \ x \in X\}$ where $\alpha + \beta \in [0,1]$.

2) $\lfloor_{\alpha,\beta} A = \{< x, \alpha\mu_A(x) + \beta, \alpha\nu_A(x) >: \ x \in X\}$ where $\alpha + \beta \in [0,1]$.

The concepts of the modal operators are being introduced and studied by different researchers, [3-6], [9, 10, 11], etc.

In 2006, the third extension of the above operators was studied by Atanassov. He defined the following operators in [4]

Definition 1.5. [4] Let X be a set, A={<x,μA (x), A (x)>: x X} IFS(X) and

$\alpha,\beta,\gamma\in[0,1]$, $\max\{\alpha,\beta\}+\gamma\le 1$. The sets $\lfloor_{\alpha,\beta,\gamma}(A)$ and $\lfloor_{\alpha,\beta,\gamma}(A)$ are defined as follows:

1) $\lfloor_{\alpha,\beta,\gamma}(A)=\{<x,\alpha\mu_A(x),\beta\nu_A(x)+\gamma>:\ x\in X\}$

2) $\lfloor_{\alpha,\beta,\gamma}(A)=\{<x,\alpha\mu_A(x)+\gamma,\beta\nu_A(x)>:\ x\in X\}$

If we choose $\alpha=\beta$ and $\gamma=\beta$ in above operators then we can see easily that

$\lceil_{\alpha,\alpha,\gamma}=\lceil_{\alpha,\beta}$ and $\lfloor_{\alpha,\alpha,\gamma}=\lfloor_{\alpha,\beta}$. Therefore, we can say that $\lceil_{\alpha,\beta,\gamma}$ and $\lfloor_{\alpha,\beta,\gamma}$ are the extensions of the operators

In 2007, the author [7] defined a new operator and studied some of its properties. This operator is $E_{\alpha,\beta}$, and defined as follows:

Definition 1.6. [7] Let X be a set and A={<x,μA (x), A (x)>: x X} IFS(X), , [0,1]. We define the following operator:

$$E_{\alpha,\beta}(A)=\{<x,\beta(\alpha\mu_A(x)+1-\alpha),\alpha(\beta\nu_A(x)+1-\beta)>:\ x\in X\}$$

If we choose =1 and write instead of we get the operator . Similarly, if =1 is chosen and writen insteed of , we get the operator .

These extensions have been investigated by several authors [10], [5,6]. In particular, the authors have made significant contributions to these operators. In 2007, Atanassov introduced the operator $_{\alpha,\beta,\gamma,\delta}$ which is a natural extension of all these operators in [5].

Definition 1.7. [5] Let X be a set, A IFS(X) $\alpha,\beta,\gamma,\delta$ [0,1] such that max $(\alpha,\beta)+\gamma+\delta\le 1.$

The operator $\alpha,\beta,\gamma,\delta$ defined by $|_{\alpha,\beta,\gamma,\delta}=\{<x,\alpha\mu_A(x)+\gamma,\beta\nu_A(x)+\delta>:\ x\in X\}$ In 2008, Atanassov defined this most general operator $\lfloor_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}$ as following:

Definition1.8. [6] Let X be a set, A IFS(X), $\alpha,\beta,\gamma,\delta,\varepsilon,\zeta\in[0,1]$ such that $\max(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\le 1$ and $\min(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\ge 0$. Then the operator $\lfloor_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}$ defined by

$$\lfloor_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}(A)=\{<x,\alpha\mu_A(x)-\varepsilon\nu_A(x)+\gamma,\beta\nu_A(x)-\zeta\mu_A(x)+\delta>:\ x\in X\}$$

In 2010, the author [8] defined a new operator as follows:

Definition 1.9. [8] Let X be a set and A={<x,μA (x), A (x)>: x X} IFS(X), , , [0,1]. We define the following operator:

$$Z^{\omega}_{\alpha,\beta}(A)=\{<x,\beta(\alpha\mu_A(x)+\omega-\omega.\alpha),\alpha(\beta\nu_A(x)+\omega-\omega.\beta)>:\ x\in X\}$$

In 2013 author defined the following operator which is a generalization of $Z^{\omega}_{\alpha,\beta}$.

Definition 1.10. [8] Let X be a set and A={<x,μA (x), A (x)>: x X} IFS(X), , , , [0,1]. We define the following operator:

$$Z^{\omega,\theta}_{\alpha,\beta}(A)=\{<x,\beta(\alpha\mu_A(x)+\omega-\omega.\alpha),\alpha(\beta\nu_A(x)+\theta-\theta.\beta)>:\ x\in X\}$$

The operator $Z^{\omega,\theta}_{\alpha,\beta}$ is a generalization of $Z^{\omega}_{\alpha,\beta}$, and also, E $_{\alpha,\beta}$, $\lceil_{\alpha,\beta}$ and $\lfloor_{\alpha,\beta}$

Before defining new operators which are d type modal operators, we will recall definitions of second type modal operators.

Definition 1.11. [2] Let X be universal and A IFS(X), [0,1]. The set D(A) defined as follows:

$$D_{\alpha}(A)=\{<x,\mu_A(x)+\alpha\pi_A(x),\nu_A(x)+(1-\alpha)\pi_A(x)>:\ x\in X\}$$

Definition 1.12. [2] Let X be universal and A IFS(X),

$\alpha,\beta\in[0,1]$ and $\alpha+\beta\leq1$. The set $F_{\alpha,\beta}(A)$ efined as follows:

$$F_{\alpha,\beta}(A)=\{<x,\mu_A(x)+\alpha\pi_A(x),\nu_A(x)+\beta\pi_A(x)>:\ x\in X\}$$

Definition 1.13. [2] Let X be universal and A IFS(X),

$\alpha,\beta\in[0,1]$. The set $G_{\alpha,\beta}(A)$ defined as follows:

$$G_{\alpha,\beta}(A)=\{<x,\alpha\mu_A(x),\beta\nu_A(x)>:\ x\in X\}$$

Definition 1.14. [2] Let X be universal and A IFS(X), $\alpha,\beta\in[0,1]$. The following sets are defined;

1) $H_{\alpha,\beta}(A)=\{<x,\alpha\mu_A(x),\nu_A(x)+\beta\pi_A(x)>:\ x\in X\}$

2) $H^*_{\alpha,\beta}(A)=\{<x,\alpha\mu_A(x),\nu_A(x)+\beta(1-\alpha\mu_A(x)-\nu_A(x))>:\ x\in X\}$

3) $J_{\alpha,\beta}(A)=\{<x,\mu_A(x)+\alpha\pi_A(x),\beta\nu_A(x)>:\ x\in X\}$

4) $J^*_{\alpha,\beta}(A)=\{<x,\mu_A(x)+\alpha(1-\mu_A(x)-\beta\nu_A(x)),\beta\nu_A(x)>:\ x\in X\}$

5) $(A)=\{<x,\mu_A(x),1-\mu_A(x)>:\ x\in X\}$

6) $\Diamond\ (A)=\{<x,1-\nu_A(x),\nu_A(x)>:\ x\in X\}$

After these studies the diagram of all modal operator a is given as following;

2. NEW OPERATORS $\lceil^{\omega}_{\alpha,\beta}$, $\lfloor^{\omega}_{\alpha,\beta}$, $E^{\omega,\theta}_{\alpha,\beta}$, $B_{\alpha\beta}$ AND $\{_{\alpha\beta}$

Definition 2.1. Let X be a universal, A IFS(X)and. We define the following operators: $\alpha,\beta,\omega\in[0,1]$.

1) $\lceil^{\omega}_{\alpha,\beta}(A)=\left\{\left\langle x,\beta(\mu_A(x)+(1-\alpha)\nu_A(x)),\alpha(\beta\nu_A(x)+\omega-\omega\beta)\right\rangle\ :\ x\in X\right\}$

2) $\lfloor^{\omega}_{\alpha,\beta}(A)=\left\{\left\langle x,\beta(\alpha\mu_A(x)+\omega-\omega\alpha),\alpha((1-\beta)\mu_A(x)+\nu_A(x))\right\rangle\ :\ x\in X\right\}$

It is clear that; $\lceil^{\omega}_{\alpha,\beta}$, $\lfloor^{\omega}_{\alpha,\beta}$ are IF operators.

From this definition, we get the following new diagram which is the extesion of the last diagram of intuitionistic fuzzy operators on IFSs in Figure 2.

Now we present some fundamental properties and relationships of new operators.

Theorem 2.1. Let X be a universal, A IFS(X)and $\alpha,\beta,\omega\in[0,1]$.

1) If $\beta\le\alpha$ then $\lceil^{\omega}_{\alpha,\beta}(\lceil^{\omega}_{\beta,\alpha}(A))\subseteq\lceil^{\omega}_{\beta,\alpha}(\lceil^{\omega}_{\alpha,\beta}(A))$

2) If $\beta\le\alpha$ then $\lfloor^{\omega}_{\alpha,\beta}(\lfloor^{\omega}_{\beta,\alpha}(A))\subseteq\lfloor^{\omega}_{\beta,\alpha}(\lfloor^{\omega}_{\alpha,\beta}(A))$

Proof (1) If we use $\beta\le\alpha$ then we get,

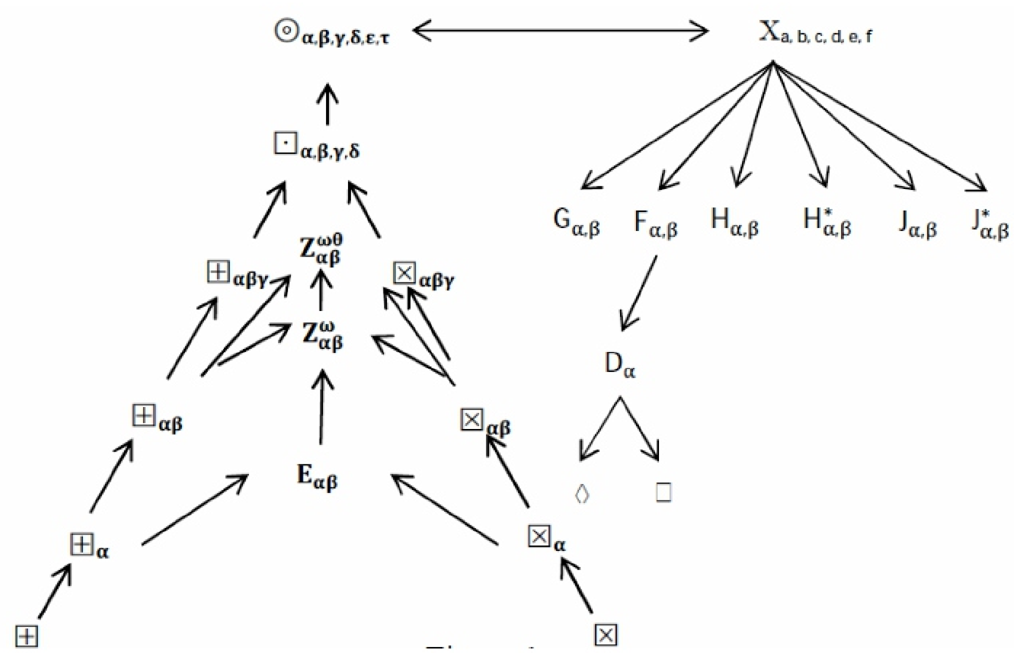

Figure 1:

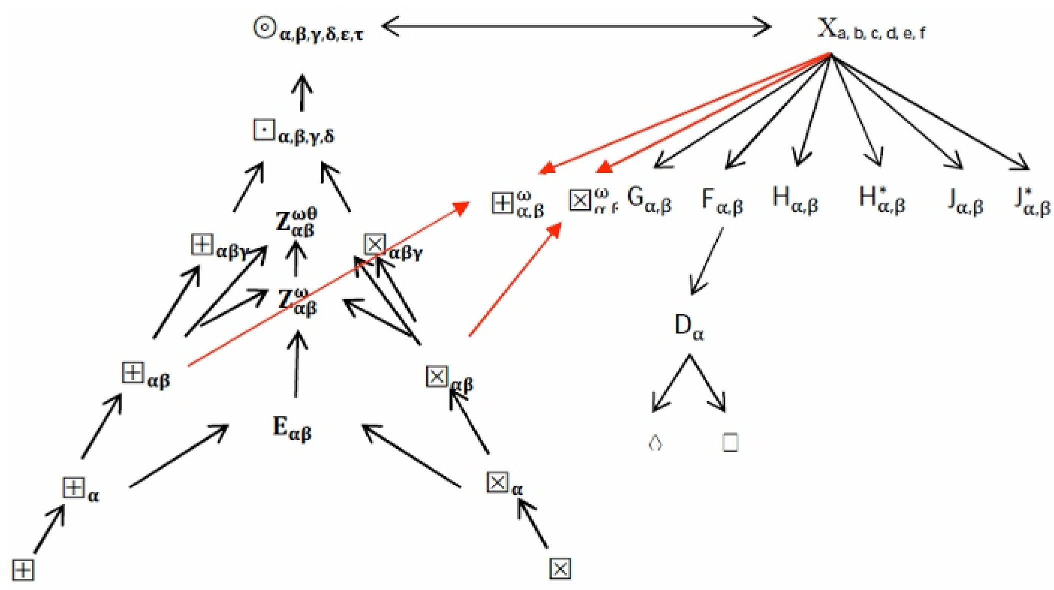

Figure 2:

$$\begin{aligned}
&\beta \le \alpha \Rightarrow (\beta-\alpha)(\beta+\alpha+2\alpha\beta) \le 0\\
&\Rightarrow \beta^2(1+2\alpha) \le \alpha^2(1+2\beta)\\
&\Rightarrow \beta^2(1+2\alpha)\omega \le \alpha^2(1+2\beta)\omega
\end{aligned}$$

and with this inequality we can say

$$\alpha\beta\mu_A(x)+\alpha\beta(1-\beta)\nu_A(x)+\beta(1-\alpha)(\alpha\beta\nu_A(x)+\beta\omega-\alpha\beta\omega)$$
$$\leq\alpha\beta\mu_A(x)+\alpha\beta(1-\alpha)\nu_A(x)+\alpha(1-\beta)(\alpha\beta\nu_A(x)+\alpha\omega-\alpha\beta\omega)$$

On the other hand

$$\beta\leq\alpha\Rightarrow(\beta-\alpha)(\alpha\beta-1)\geq 0$$
$$\Rightarrow\alpha\beta^2+\alpha-\alpha\beta\geq\alpha^2\beta+\beta-\alpha\beta$$
$$\Rightarrow\alpha\beta^2\omega+\alpha\omega-\alpha\beta\omega\geq\alpha^2\beta\omega+\beta\omega-\alpha\beta\omega$$

with this we can say

$$\alpha^2\beta^2\nu_A(x)+\alpha\beta^2\omega-\alpha^2\beta^2\omega+\alpha\omega-\alpha\beta\omega\geq$$
$$\alpha^2\beta^2\nu_A(x)+\alpha^2\beta\omega-\alpha^2\beta^2\omega+\beta\omega-\alpha\beta\omega$$

So we get

$$\lceil^{\omega}_{\alpha,\beta}(\lceil^{\omega}_{\beta,\alpha}(A))\subseteq\lceil^{\omega}_{\beta,\alpha}(\lceil^{\omega}_{\alpha,\beta}(A))$$

We can show the property (2) with the same way. 2.1. Let X be a universal, A IFS(X) and $\alpha,\beta,\omega\in$ [0,1). Then the following statements hold:

1) $\lceil^{\frac{\beta}{1-\alpha}}_{1,\alpha}(A)=\lceil_{\alpha,\beta}(A)$

2) $\lfloor^{\frac{\beta}{1-\alpha}}_{\alpha,1}(A)=\lfloor_{\alpha,\beta}(A)$

Proof It is clear from definition.

Definition 2.2. Let X be a set and A= $\{<x,\mu_A(x),\nu_A(x)>:\ x\in X\}\in IFS(X)$, $\alpha,\beta,\omega,\theta\in[0,1]$.

We define the following operator:

$$E^{\omega,\theta}_{\alpha,\beta}(A)=\{<x,\beta((1-(1-\alpha)(1-\theta))\mu_A$$
$$(x)+(1-\alpha)\theta\nu_A(x)+(1-\alpha)(1-\theta)\omega),$$

$$\alpha((1-\beta)\theta\mu_A(x)+(1-(1-\beta)(1-\theta))\nu_A$$
$$(x)+(1-\beta)(1-\theta)\omega)>:\ x\in X\}$$

Proposition 2.2. Let X be a set and A IFS(X), $\alpha,\beta,\omega,\theta\in[0,1]$. $E^{\omega,\theta}_{\alpha,\beta}(A^c)=E^{\omega,\theta}_{\beta,\alpha}(A)^c$

Proof It is clear from definition.

Proposition 2.3. Let X be a set and A IFS(X), $\alpha,\beta,\omega,\theta\in[0,1]$. If $\beta\leq\alpha$ then

$$E^{\omega,\theta}_{\alpha,\beta}(A)\subseteq E^{\omega,\theta}_{\beta,\alpha}(A)$$

Proof If we use $\beta \le \alpha$ then

$$\beta \le \alpha \Rightarrow \beta(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta)) \le \alpha(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta))$$
$$\Rightarrow \beta(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta))+\alpha\beta(\mu_A(x)+\theta\mu_A(x)-\theta\nu_A(x))$$
$$\le \alpha(\theta(\mu_A(x)+\nu_A(x))+\omega(1-\theta))+\alpha\beta(\mu_A(x)+\theta\mu_A(x)-\theta\nu_A(x))$$

Then we can say $E^{\omega,\theta}_{\alpha,\beta}(A) \subseteq E^{\omega,\theta}_{\beta,\alpha}(A)$.

Proposition 2.4. Let X be a set and A IFS(X) $\alpha,\beta,\omega,\theta \in [0,1]$. If $\omega \le \theta$ then

$$E^{\omega,\theta}_{\alpha,\beta}(A) \subseteq E^{\theta,\omega}_{\alpha,\beta}(A)$$

Proof: It is clear from definition.

Definition 2.3. Let X be a set, A IFS(X) and $\alpha,\beta \in [0,1]$. We define the following operator:

$$B_{\alpha,\beta}(A) = \left\{ \begin{array}{l} \langle x, \beta(\mu_A(x)+(1-\alpha)\nu_A(x)), \alpha((1-\beta)\mu_A(x)+\nu_A(x)) \rangle \\ : x \in X \end{array} \right\}$$

Definition 2.4. Let X be a set, A IFS(X) and $\alpha,\beta,\omega \in [0,1]$ We define the following operator:

$$\{_{\alpha,\beta}(A) = \{ \langle x, \beta(\mu_A(x)+(1-\beta)\nu_A(x)), \alpha((1-\alpha)\mu_A(x)+\nu_A(x)) \rangle : x \in X \}$$

Theorem 2.2. Let X be a set, A IFS(X) and $\alpha,\beta \in [0,1]$. Then the following statements hold:

$$B_{\alpha,\alpha}(A) = \{_{\alpha,\alpha}(A)$$

Proof: It is clear from definition.

Theorem 2.3. Let X be a set, A IFS(X) and $\alpha,\beta,\omega \in [0,1]$. The following statements are satisfied:

$$1)\ \lceil^{\omega}_{\alpha,\beta}(A^{c}) = \lfloor^{\omega}_{\beta,\alpha}(A)^{c}$$

$$2)\ \lfloor^{\omega}_{\alpha,\beta}(A^{c}) = \lceil^{\omega}_{\beta,\alpha}(A)^{c}$$

$$3)\ \{_{\alpha,\beta}(A^{c}) = \{_{\beta,\alpha}(A)^{c}$$

Proof (1) From definitions of these operators and complement of an intuitionistic fuzzy set we get that,

$$\lfloor^{\omega}_{\beta,\alpha}(A)^{c} = \{< x, \beta((1-\alpha)\mu_A(x) + \nu_A(x)), \alpha(\beta\mu_A(x) + \omega - \omega\beta) >:\ x \in X\}$$

and

$$\lceil^{\omega}_{\alpha,\beta}(A^{c}) = \left\{\left\langle x, \beta(\nu_A(x) + (1-\alpha)\mu_A(x)), \alpha(\beta\mu_A(x) + \omega - \omega\beta)\right\rangle\ :\ x \in X\right\}$$

So, we can say $\lceil^{\omega}_{\alpha,\beta}(A^{c}) = \lfloor^{\omega}_{\beta,\alpha}(A)^{c}$.

(2) It is clear from definition.

(3) If we use definitions then we get

$$\{_{\alpha,\beta}(A^{c}) = \left\{\left\langle x, \beta(\nu_A(x) + (1-\beta)\mu_A(x)), \alpha((1-\alpha)\nu_A(x) + \mu_A(x))\right\rangle\ :\ x \in X\right\}$$

and

$$\{_{\beta,\alpha}(A)^{c} = \left\{\left\langle x, \beta((1-\beta)\mu_A(x) + \nu_A(x)), \alpha(\mu_A(x) + (1-\alpha)\nu_A(x))\right\rangle\ :\ x \in X\right\}$$

So, we can say; $\{_{\alpha,\beta}(A^{c}) = \{_{\beta,\alpha}(A)^{c}$

Theorem 2.5. Let X be a set and A IFS(X),

$\alpha, \beta \in [0,1]$. If $\alpha \geq \frac{1}{2}$, $\beta \leq \frac{1}{2}$ then

$$B_{\alpha\beta}(B_{\beta\alpha}(A)) \subseteq B_{\beta\alpha}(B_{\alpha\beta}(A))$$

Proof If we use 1\2 and 1\2 then we get,

$$(1-2\alpha) \le (1-2\beta) \Rightarrow \beta^2(1-2\alpha)(\mu_A(x)+\nu_A(x)) \le \alpha^2(1-2\beta)(\mu_A(x)+\nu_A(x))$$

So,

$$\alpha\beta\mu_A(x)+\alpha\beta(1-\beta)\nu_A(x)+\beta^2(1-\alpha)^2\mu_A(x)+\beta^2(1-\alpha)\nu_A(x)$$
$$\le \alpha\beta\mu_A(x)+\alpha\beta(1-\alpha)\nu_A(x)+\alpha^2(1-\beta)^2\mu_A(x)+\alpha^2(1-\beta)\nu_A(x)$$

and

$$\alpha^2(1-\beta)\mu_A(x)+\alpha^2(1-\beta)^2\nu_A(x)+\alpha\beta(1-\alpha)\mu_A(x)+\alpha\beta\nu_A(x)$$
$$\ge \beta^2(1-\alpha)\mu_A(x)+\beta^2(1-\alpha)^2\nu_A(x)+\alpha\beta(1-\beta)\mu_A(x)+\alpha\beta\nu_A(x)$$

with these inequalities $B_{\alpha\beta}(B_{\beta\alpha}(A)) \subseteq B_{\beta\alpha}(B_{\alpha\beta}(A))$.

As a consequence of above theorem, we can get easily the following propositions;

Proposition 2.5. Let X be a set and $A\,IFS(X), \alpha,\beta \in [0,1]$. Then,

$$B_{\alpha\beta}(A^c) = B_{\beta\alpha}(A)^c$$

Proposition 2.6. Let X be a set and $A\,IFS(X)$, $\alpha,\beta \in [0,1]$ Then, $\alpha,\beta,\omega \in [0,1]$. Then,

1) $E^{\omega,0}_{\alpha,\beta}(A) = Z^{\omega}_{\alpha,\beta}(A)$

2) $E^{\omega,1}_{\alpha,\beta}(A) = B_{\alpha,\beta}(A)$

3) $E^{0,0}_{\alpha,\beta}(A) = G_{\alpha\beta,\alpha\beta}(A)$

4) $E^{1,0}_{\alpha,\beta}(A) = E_{\alpha,\beta}(A)$

5) $E^{0,0}_{1,0}(A) = \varnothing$

6) $E^{0,0}_{0,1}(A) = X$

Proposition 2.7. Let X be a set and A IFS(X) $\alpha,\beta \in [0,1]$ Then,

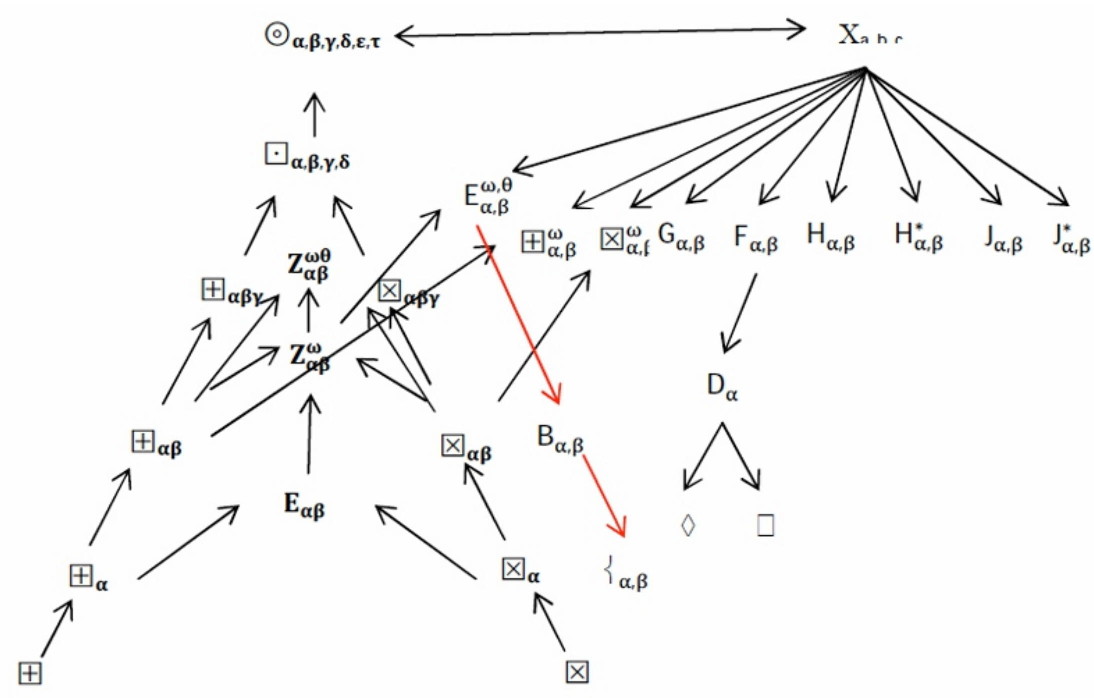

Figure 3:

1) $E^{1,0}_{\alpha,1}(A) = \lfloor_{\alpha}(A)$

2) $E^{\omega,0}_{\alpha,1}(A) = \lfloor_{\alpha,\omega,(1-\alpha)}(A)$

3) $E^{\omega,0}_{1,\beta}(A) = \lceil_{\beta,\omega(1-\beta)}(A)$

4) $E^{\omega,\theta}_{1,1}(A) = A$

5) $E^{1,0}_{1,\beta}(A) = \lceil_{\beta}(A)$

6) $E^{\omega,1}_{\alpha,1}(A) = B_{\alpha,1}(A)$

7) $E^{1,1}_{1,\beta}(A) = B_{1,\beta}(A)$

- From the above properties, it is easily show that the operators $\lceil^{\omega}_{\alpha,\beta}$, $\lfloor^{\omega}_{\alpha,\beta}$, $E^{\omega,\theta}_{\alpha,\beta}$, $B_{\alpha\beta}$ and $\{_{\alpha\beta}$ which are defined in this paper are both one type and two type operators. From the above discusssion and the common properties of these operators with the one and two type operators, we give the following definition for the classification;

Definition 2.5. Let X be a set and be a modal operator of Intuitionistic Fuzzy Set on X. If is both one type and two type modal operator then it is called uni-type modal operator of Intuitionistic Fuzzy Set on X.

From that fundemental properties we get the last diagram of all (one/two/uni-type) modal operators on Intuitionistic Fuzzy Sets as in Figure 3;

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On Jump-Critical Ordered Sets with Jump Number Four

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Abstract:

For an ordered set P and for a linear extension L of P, let s(P,L) stand for the number of ordered pairs (x, y) of elements of P such that y is an immediate successor of x in L but y is not even above x in P. Put s(P) = min {s(P, L): linear extension of P}, the jump number of P. Call an ordered set P jump-critical if s(P - {x}) < s(P) for any x P. We introduce some theorems about the jump-critical ordered sets with jump number four.

Keywords: Jump number, jump-critical ordered sets, tower poset.

1. INTRODUCTION

Let P be a poset and L be a linear extension of P. Every linear extension L of a finite ordered set P can be expressed as the linear sum C1 C2 … Cm of chains Ci of P so labeled that supP Ci infP Ci+1 in (The linear sum A B of ordered sets A and B is the set A B ordered so that a b provided that a A and b B, or else, a b in A or, a b in B). Let Ci = {ai = ai1 < ai2 < …< aiki = bi}. Then bi ai+1 in P and such a pair (bi, ai+1) is called a jump (or set up) of the linear extension L, which is said to have m-1 jumps. We write s (P, L) = m-1. Note that ai+1 covers bi in L, although ai+1 bi in P itself. We put s(P) = min {s (P,L)| L linear extension of P}. This problem finds its practical settings too. Let the elements of P represent certain jobs to be performed one at a time by a single processor while the order of P imposes precedence constraints upon these jobs. Then an optimal linear extension of P is just a schedule of the jobs which minimizes the number of " set up" between unrelated jobs.

Observe that s(P) > s(P - {x}) > s(P) - 1 for any x P. A poset P is called jump-critical if s (P – x) < s (P), for every element x P. If P is jump-critical with s(P) = m, then we say that P is m-jump-critical. It is easy to see that every ordered set P contains a jump-critical subset K with s (P) = s (K). It may be that jump-critical ordered sets tell us much about the problem determining s(P) - even about constructing "optimal" linear extensions for P, that is, ones for which s(P, L) = s(P). The ordered set illustrated in Figure 1 is jump-

s(P - {a31}) < 4, for instance, requires a different chain decomposition: P - {a31} = C2 C4 C5 {a11 < a12 < a32}. It is a good exercise to check all eight cases.

The purpose of this paper is to stimulate activity on the jump number of an ordered set by recording several important examples. In section 2, we introduce some special methods to construct jump-critical ordered sets. In section 3, we introduce the complete lists of 1-jump-critical, 2-jump-critical, 3-jump-critical ordered sets and some theorems about 4-jump-critical ordered sets.

2. SPECIAL METHODS TO CONSTRUCT JUMPCRITICAL ORDERED SETS

In this section we present special methods for constructing jump-critical posets. An n-element antichain is jump-critical. In fact, it is fairly obvious that the disjoint sum of jump-critical ordered sets is jumpcritical. In addition, s(P1 + P2) = s(P1) + s(P2) + 1. It is equally obvious that the linear sum of jump-critical ordered sets is jump-critical. Also s(P1 P2) = s (P1) + s (P2). These are special cases of a more general construction. Let P be an ordered set and each x P, let x P be an ordered set. The lexicographic sum

$\sum_{x\in P} P_x$ is the set $\bigcup_{x\in P} P_x$ ordered so that $u \leq v$ if, for some $x \in P$, $u \in P_x$, $v \in P_x$ and $u \leq v$ in P_x, or else, $u \in P_x$, $v \in P_y$, for some $x < y$ in P. It is implicit in M. Habib [5] that the lexicographic sum $\sum_{x\in P} P_x$ of critical

ordered sets Px is itself critical, as long as each | Px |>2. M. H. El-Zahar and I. Rival introduced a new method which gets jump-critical ordered sets by the theorem 1 [2].

Theorem 1: Let P1 and P2 be finite jump-critical ordered sets. Any ordered set obtained from P1 and P2 by gluing a maximal element of P1 with a maximal

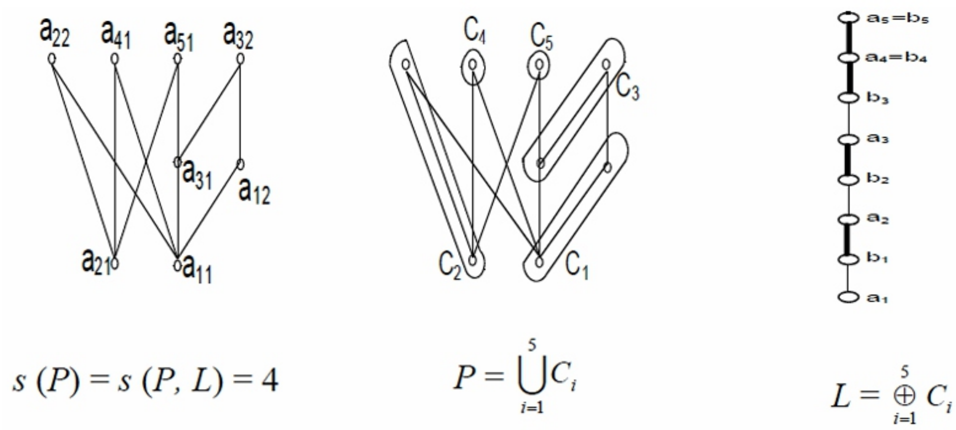


Figure 1:

element of P2 is jump-critical and, in this case, the jump number is s(P1) + s(P2). If | max P1 | = | max P2 | =2 then the ordered set obtained from P1 and P2 by gluing max P1 with max P2 is jump-critical and, in this case, the jump number is s(P1) + s(P2) -1.

This gluing construction can be used to construct an example of jump-critical ordered set in which an "optimal" linear extension uses a long chain (see Figure 2).

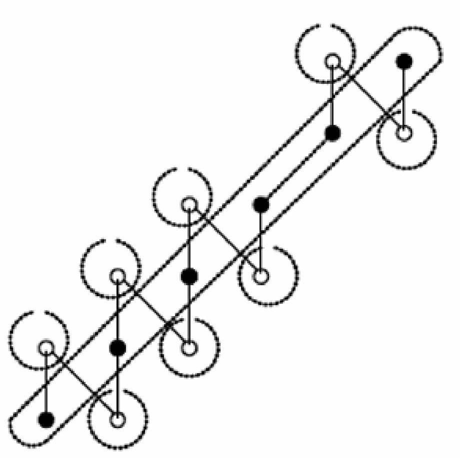


Figure 2:

There is an obvious question that arises for the second part of Theorem 1: does the gluing construction produce a jump-critical ordered set if there are more than two maximal elements? This question is open until now.

3. (1-4) JUMP-CRITICAL ORDERED SETS

In this section, we introduce the complete lists of 1jump-critical, 2-jump-critical, 3-jump-critical ordered sets and some theorems about 4-jump-critical ordered sets. Obviously, the only jump-critical ordered sets P with s(P) = 0 is the singleton. If s(P) = 1 then, of course, P must contain a noncomparable pair of elements. So, if P is jump-critical then P must be a two-element

antichain. Suppose P is jump-critical and s(P) = 2. P may be a three-element antichain. The only other possibility is that P is the " four-cycle", as showed in Figure 3. Thus, either P 1 + 1 + 1 or P (1 + 1) (1 + 1).

M. H. El-Zahar and I. Rival [2] introduced the complete list of the jump-critical ordered sets with jump number three which has fourteen jump-critical ordered sets. These are, up duality, the ordered sets illustrated in Figure 4.

Let P be a finite ordered set. For an element a in P put D(a) = { x P|x a}, a down set in P, U(a) = { x P|x a}, an upper set in P. Following M. H. El-Zahar and J. H. Schmerl [3] call the element a accessible in P if D(a) is a chain in P. For instance, each minimal element of P is accessible. Let w(P) stand for the width of P, the size of a maximum-sized antichain. According to Dilworth's chain decomposition theorem (1), P is the (disjoint) union of w(P) chains. For maximum-sized antichains A, B in P we write A B whenever for a A there is b B satisfying a b. (It follows, in this case that, for each b B there is a A satisfying a b, too). In this way the set of maximumsized antichains of P is ordered: there is greatest (highest) antichain and a least (lowest) antichain. As matter of fact, the set of maximum-sized antichains is a distributive lattice in which A B=max(A B) and A B=min(A B) (R. P. Dilworth [1]). A tower of height k (or k-tower) is a linear sum of k-comparable elements [4]. Obviously, a k-tower is k-critical with width two.

Theorem 2. Let P be a k-jump-critical ordered set with width 2 where k >1. Then P is a k-tower.

Proof: We use induction on k. For k = 2, the only poset which satisfies the criteria of the theorem is the 4-alternating-cycle 2 2. Thus, the result is true for k =

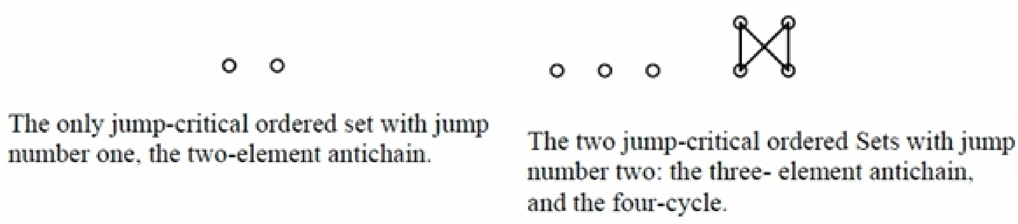


Figure 3:

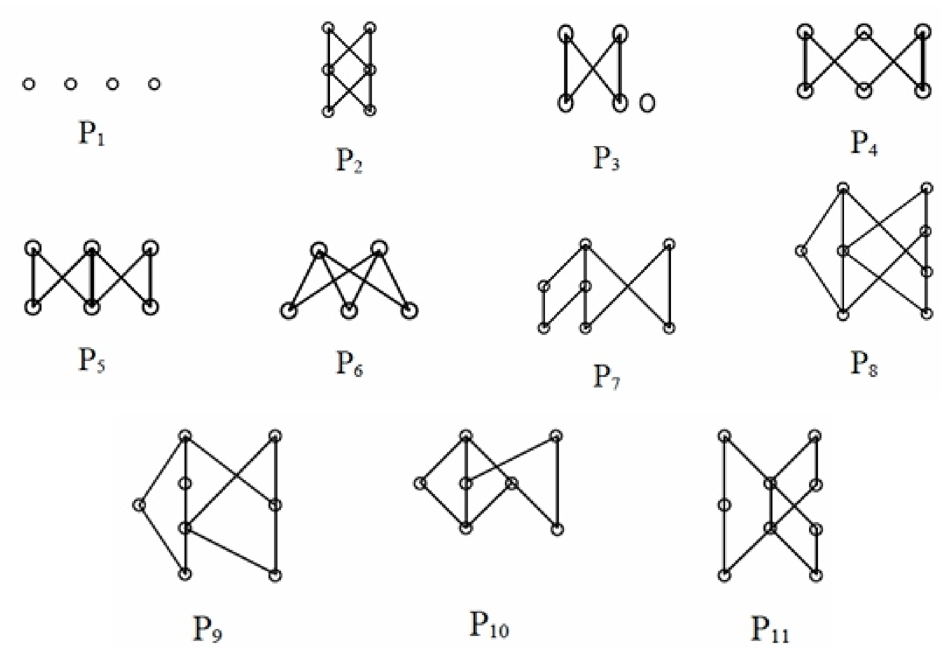


Figure 4:

2, and assume that it is true for jump-critical posets with jump-number less than k. Now we want to prove that it is true for jump-critical posets with jump-number k.

Since w(P) = 2 then it is the union of two chains C1 and C2. Put xi = infp Ci for i = 1,2. As P is jump-critical then x1 x2. Let ai be the maximal accessible element on Ci ; i = 1,2. See Figure 5.

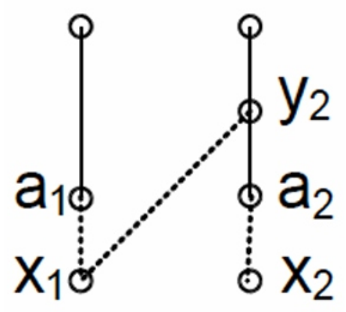
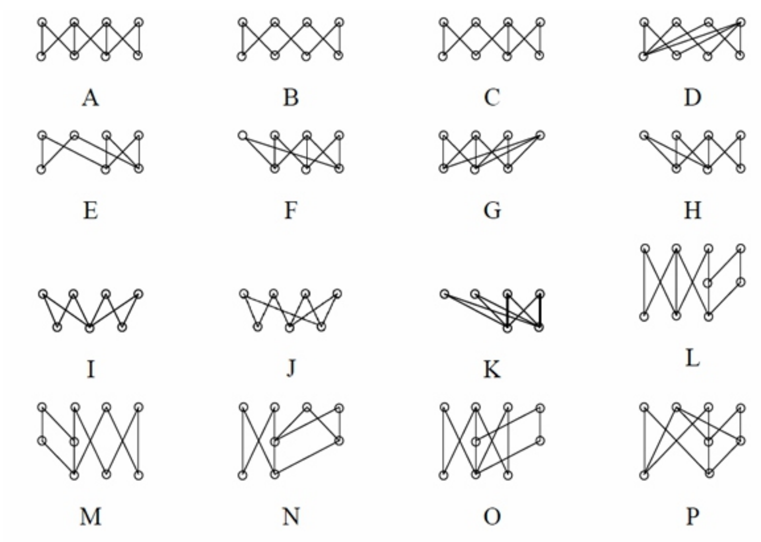


Figure 5:

We want to prove that ai = xi, i = 1,2. Suppose not, say a1 > x1. Put P` = P - {a1}. As P is jump-critical then s (P`) = k -1. Let L be a linear extension of P` with k-1 jumps, say L = C1` .
= 1, 2, k. If x1 C`k where each C`i is a chain, i C`1 then C`1 {a1} is also a chain. So,
we can replace C`1 on L by C`1 {a1} which gives a linear extension of P with only k-1 jumps. This a contradiction. So, x1 C`1 which implies that C`1 = x2 a2. Now x1 C`2. If C`2 C2 = then C`2 chain. Again, we can replace C`2 by C`2 {a1} is a {a1} to get s (P) = k-1; a contradiction. Therefore C`2 has the form C`2 = x1 … y2 … m where y2 is the element that covers a2 on C2
m = max C`2 is some element in C2 (possibly m = y2). Now we can replace C`1 and C`2 respectively by C``1 and C``2 where C``1 = x1… a1 and C``2 = x2 …a2 y2 … m. This gives a linear extension of P with only k-1 jumps which is a contradiction. We conclude that a1 = x1 and similarly a2 = x2. Now P - {x1, x2} has jump number k-1 and, by induction, contains a (k-1) tower. This (k-1) tower together with {a1, a2} forms a k-tower. This must be all of P. This completes the proof of the Theorem.

Theorem 3. There are precisely forty jump-critical ordered sets with four maximal elements and s(P) = 4. These are, up duality, the ordered sets illustrated in Figure 6.

Proof of Theorem 3. It is straightforward, if



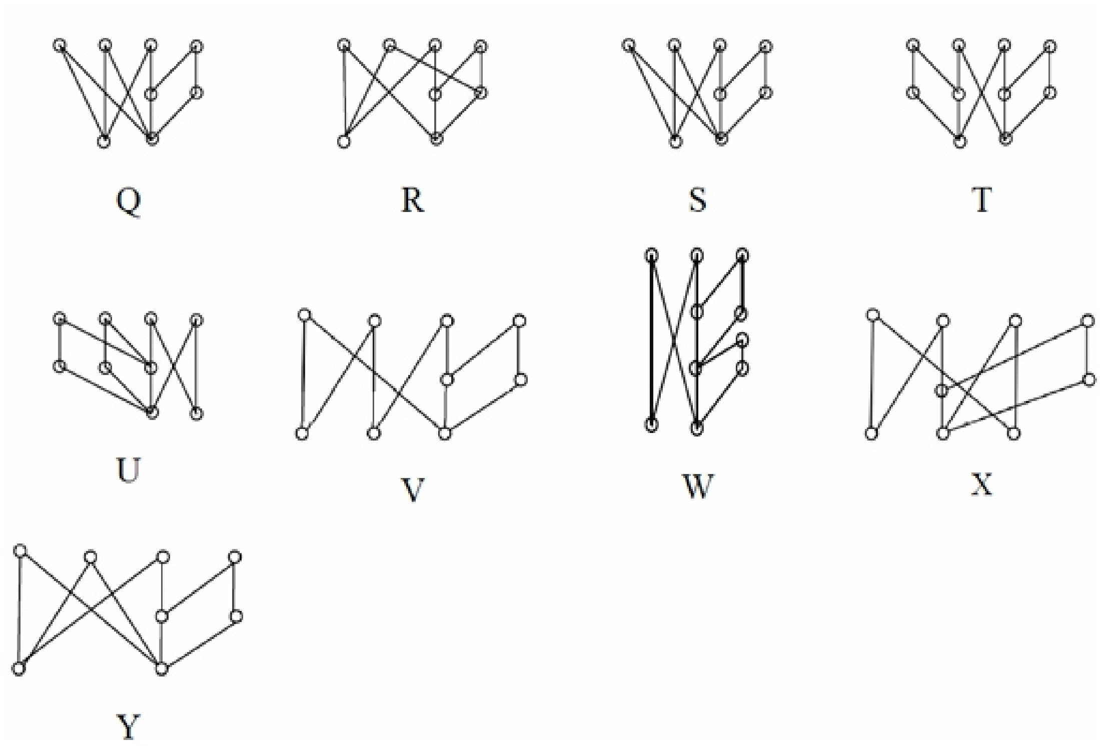


Figure 6:

somewhat laborious, to verify that each of the ordered sets illustrated in Figure 6 has jump number four, four maximal elements and that each is jump critical without isolated element.

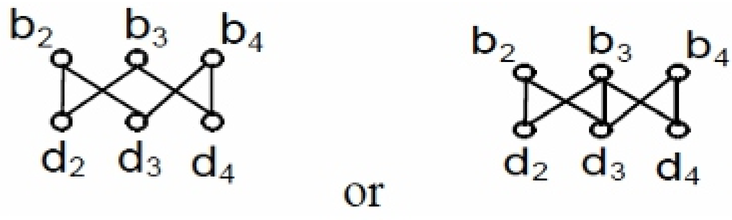
Let P be 4-jump-critical and has four maximal elements (without isolated element). For contradiction, suppose that P contains no subset isomorphic to any of the posts illustrated in Figure 6.

Since P is 4-jump-critical with w(P) = 4,then P = C1 C2 C4 (disjoint chains). Put ai = infP Ci and bi = supP Ci for i = 1, 2, 3, 4. Let us suppose that both {a1, a2, a3, a4} is an antichain and {b1, b2, b3, b4} is maximal elements antichain. If bi 's is accessible, then ai b2, b3, b4} contains E (Ed) or F(Fd) or G(Gd) or H(Hd) or

J(Jd). Next, we handle the case {a1, a2, a3, a4} is not antichain. Let {c1, c2, c3, c4} be infimum of all fourelement antichain in P.

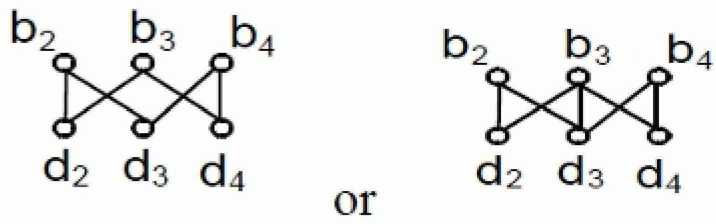
One of ci's must be less than one of bi's, only, say c1 4 < b1, for otherwise the proper subsets (U[ci] has jump number four. If (P) of P i=1 U[c1]) contains fourelement antichain, {x1, x2, x3, x4} then c1 must be comparable to one of these xi's (say) x1. But x1 > c1, since x1 U(c1) and if bi, | D[bi] {a1, a2, a3, a4}| > b3, b4}| > 2 and, dually, | D[ai] {b1, b2, 2. If follows that {a1, a2, a3, a4, b1, b2, b3, b4} is isomorphic to A or B or C or D. Or that {a1, a2, a3, b1, x1 < c1 then {c1, c2, c3, c4} is not the lowest fourelement antichain in P. Therefore, w (P U[c1]) = 3

and we can assume that, (P U[c1]) = C2 C3 C4 so that U[c1] = C1. Let {d2, d3, d4} and {b2, b3, b4} be respectively, the lowest and highest, three-element antichain in C2 C3 C4 where, say, di, bi Ci for both i = 2, 3, 4 since s (C2 C3 C4 = 3) then {d2, d3, d4, b2, b3, b4} is isomorphic to the following posets



Neither bi is above c1. Also c1 can not below di 's, otherwise c1 < one of bi 's only. Moreover c1 > d2 or c1 > d3 or c1 > d4. Otherwise c1 is an isolated element in P. Therefore min (P) = min (C2 C3 C4). For otherwise P would have a unique minimal element.

If c1 > d2, c1 > d3 and c1 > d4 then
{c1, d2, d3, d4,b2, b3, b4} E or {c1, d2, d3, d4,b2, b3,
b4} G.
If c1 > the two elements of {d2, d3, d4} then
{c1, d2, d3, d4,b2, b3, b4} E or {c1, d2, d3, d4,b2, b3,b4} H.
We may then suppose that c1 > d2, c1 > d3 and c1 >
d4. Since b1 = supP C1, let us suppose that b1 > b2, b1 > b3 and b1 > b4 then there exists an element d C2 C3 C4 such that d d2, d < b1 and c1 || d. Otherwise, c1 is an accessible in the Pd, as (P U[c1]) has width three and jump number three so it must contain



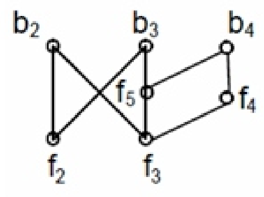
So that any of these Figures, with ci is a subposet of P contains isolated element c1. If b1> one of {d3, d4} then {b1,b2,b3,b4,d2,d3,d4} F or H. Otherwise

(i) $d_2 < d < b_2$, $d \,||\, b_3$ and $d \,||\, b_4$ or

(ii) $d_3 < d < b_5$, $d \,||\, b_2$, $d \,||\, b_4$ and $d \,||\, b_1$

If (i) satisfies then { b1, c1, d, b2, b3, b4, d2, d3, d4 } is isomorphic to L, M, V or X; if (ii) satisfies then { b1, c1, b2, b3, b4, d2, d3, d4, d} is isomorphic to U. Now let {f2, f3} and {b2, b3, b4} be, respectively, the lowest and height, two-element antichain and three-element antichain in

$C_2 \cup C_3 \cup C_4$ where f_i, $b_i \in C_i$ for i = 2, 3, 4. Since $s(C_2 \cup C_3 \cup C_4)$ = 3 then P contains



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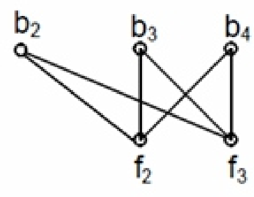
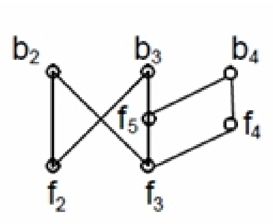
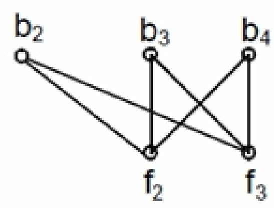


Neither bi is above ci. Also ci can't below fi 's otherwise, c1 < one of bi 's only. Moreover c1 > f2 or c1 > f3 or c1 > f4 or c1 > f5 otherwise c1 is isolated element in P. Therefore min (P) = min (C2 C3 C4), for otherwise P would have a unique minimal element. If c1 > f2 and c1 > f3, then {c1, b2, b3,b4, f2, f3} K. If c1 > f2 and c1 > f4 and c1 > f5; since c1 || b2, c1 || and c1 || b4 then {c1, b2, b3, b4, f2, f3, f4, f5} P. If c1 > f2, c1 > f4 and c1 || f5 since c1 || b2, c1 || b3 and c1 || b4 then {c1, b2, b3, b4, f2, f3,f4,f5} R, O or Y. Now, if c1 > f2 and c1 > f3; since b1 = supP C1, let us suppose that b1 > b2, b1 > b3 and b1 > b4 then there is an element f C2 C3 C4 such that f f2; f < b1 and c1, f are incomparable, otherwise c1 is an accessible in the Pd. As (P U[c1]) has width three and jump number three, it must contains



or



So that any of these Figures with c1 is a subposet of P contains isolated element c1. If b1 > f3 then {b1, b2, b3,b4, f2, f3} K and {b1, b2, b3,b4, f2, f3,f4,f5} S otherwise f2 < f < b2 and f || b3 and f || b4 then {c1, f, b1, b2, b3, b4, f2, f3,f4, f5} T. If f3 c1 or f4 c1 or f5 c1 and c1 > f2; since f5 f3, f5 > f3 therefore f5 || c1,

otherwise $P-(\bigcup_{i=1}^{4} U[c_i])$ has jump number four. Then, if

b1 > f5 then {b1, b2, b3, b4, c1, f2, f3, f4,f5} W, if f4 c1, b > f and f > f4 then {b1, b2, b3, b4, f2, f3, f4,f5, f, c1} V, if f5 c1, c1 > f4 and c1 > f2 then { c1, f2, f3, f4,f5, b2, b3, b4} N and if f5 c1, f5 f, b1 > f then {b1, b2, b3, b4, f2, f3, f4, f5, f, c1} W. Hence this theorem is proved.

CONCLUSION

In this paper, we introduced some theorems about 4-jump-critical ordered sets. In future, we can investigate the structure of m-jump-critical ordered sets to study the jump-number problem.

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Quantum Game Techniques Applied to Wireless Networks Communications

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Abstract:

In order to analyze the power control problem, the wireless quantum network nodes are modeled as players at a quantum game. The power control problem is one of the most significant wireless communications challenges which characteristics make it proper to be modeled by means of game theory techniques. The problem results in noncooperative game by nature, but, under quantum rules, a larger strategy space leads the players to choose a coalition strategy as the best option. Thus, the use of quantum game strategies makes possible the emergence of new equilibrium, which guarantees the best possible performance to the whole network. We show also that the whole network power consumption decreases when the intrinsic parallel behavior of quantum computation is capitalized. Moreover, the design of efficient medium access control algorithms is possible.

Keywords: Game theory, Quantum Computation, Wireless Communications.

1. INTRODUCTION

In the last decades, there has been a breakthrough in wireless communication networks. Many types of portable communication devices, such as

smartphones, tablets, PDAs are carried by many people for use in the different domains of their lives. Thus, given the plenty of transmission protocols and software radio capabilities, networks are evolving to less structured and increasingly involve distributed decision making. Power control problems games are about the right amount of power the nodes of a network must use to send information through the available channels.

When the used power increases, the wireless devices battery life diminishes and the interference between users increases. On the other hand, there is a minimum of transmission power that satisfies the quality of service thresholds. In other words, from a particular user point of view, an efficient power control algorithm must support him with some minimum acceptable throughput, whereas from the whole network point of view, the aggregate throughput must be maximized. Accordingly, many techniques have proven to be successful by various authors and researchers for this purpose. Among these techniques, because of the problem characteristics, the most appropriate are based on game theory models.

The application of game theory dates back to the 90's. Game theory is the study of strategic decision making, where the decision makers are players whose utilities (or payoffs) depend on other players' in a game [3-9], so that they compete or even cooperate in order to achieve the wanted quality of service.

In [10] for instance, the authors studied the competitive and cooperative distributed spectrum coordination techniques for the two users Gaussian interference game. The author shows that the most used IWF (Iterative water-filling) algorithm is not optimum. The IWF algorithm converges to a situation in which the power of one transmitter is allocated uniformly in every possible channel [11, 12], however a problem arises when more than one transmitter are involved. Because of the competing interests, this situation can lead to the prisoner's dilemma [13]. The prisoner's dilemma is about two persons who are arrested and put in separated rooms to be interrogated. The police talk to them and tell their options: If they both cooperate with each other (do not confess) they receive a minimum sentence, three months for example. If only one of them betrays (confess) this one is freed but the other is considered guilty of all the charges and given the maximum penalty, five years for example. On the other hand, if both betray and plead guilty, they receive a sentence of 1 year in jail. Thus, each prisoner must decide between to cooperate which would benefit both, but at risk of being betrayed or to betray in order to protect himself. As both of them receive the same deal, both decide to betray although through cooperation they could serve less prison time. Therefore, the problem has a Nash equilibrium, which is not Pareto optimum, i.e., users do not achieve the maximum rate. On the contrary, the quantum version of the prisoner's dilemma game does have an optimum solution [14]. In this framework, the quantum model presented here is game based model where players take their decisions thinking about not only but also on the others benefits in order that the whole network gets the best possible performance. In this way, quantum games larger space of strategies gives the players new chances inducing the system to new stability points.

In this paper, we present an N players interference quantum model and analyze players' performance of using classic or quantum strategies. Because of the network users must decide between the whole network health and their own, the prisoner's dilemma is also present in this case. However, the dilemma can be eliminated by means of quantum entanglement and quantum superposition, two features only feasible under quantum computation. Then, it is possible to consider a quantum phase, interleaved in the real classic protocols, that manages fairly the users' power spectrum.

2. A QUANTUM GAME OF INTERFERENCE

Power control becomes necessary when a set of wireless mobiles share a common network. The purpose is to let every user to send information without causing unnecessary interference to the others. That is, the most power they use, the harder the interference they can cause to the neighbors receptions. Besides, the more power they save, the longer is the wireless devices battery life. On the other hand, because there is a constraint in the minimum of transmission power necessary to satisfy

Hereinafter, we define the main aspects of the classic problem, after that, we will explain the necessary changes to build the quantum model and the advantages that it brings with.
In this model, as in the real scenario, N independent network users with no information about other user actions are considered. They are free to choose among N channels, so they can use only one (cooperative attitude), otherwise they can distribute their power among all the available channels (selfish attitude). It is supposed that every user may transmit a total power P, that is, in one channel or distributed through all the channels. The payoffs obtained by the players are linked to the Signal-to-interference-and-noise-ratio (SINR), that is, the greater the SINR the better is the reception quality. The signal S is related to the power used by one player, the interference I is related to the power received from the other players. As a consequence, the best way to assign value to the reward (payoff) some player j obtains from the network is through the Shannon Capacity, [15], given by equation (1), where j kP is the power that transmitter j assigns to channel k and hij (k) is the k channel gain between the transmitter jth and receiver ith . Moreover, (k) is the thermal noise at the receiver on channel k.

$$C_j = \sum_{k=1}^{N} \log_2 \left(1 + \frac{h_{ij}(k).\alpha_j^k P}{\sigma(k) + \sum_{i \neq j}^{N} \alpha_j^k P.h_{ij}(k)} \right) \qquad (1)$$

The players strategies consist on choosing the fractions of P assigned to each channel by means of Aj =[j 1, j 2,… j N], thus j iP denotes, for instance, the portion of power player j assigns to channel I. Thereby, the payoff each player receives will depend on the quality of the channel and the chosen action vector A.
In our model, the players must decide between two extreme options, these are Cooperating, which implies to select only one channel, or Defecting, which implies to distribute the power among all the channels. It is known that this problem has an inefficient solution classically, since the users finds to Defect as the best option. However, we show that it is possible to quantize the model in order to improve the players utilities. Consequently, in what follows we describe the characteristics of the quantum model.

$$|S0\rangle = \frac{|00\ldots0\rangle + i|11\ldots1\rangle}{\sqrt{2}} \qquad (2)$$

In the first place, the system initial state S0 depicted in (2) corresponds to the quantum superposition of the every user cooperating state, 00…0 and the every user defecting state, 11…1 . So, note that “0” in some position j means that user 0 j N 1 decides to cooperate and a “1” means the opposite situation. Note also that without the users intervention, the system outcome can only be one of this two situations with probability 1/2. Every user is aware about the initial state and they are able to transform the system final state according to their strategies. The users strategies must transform the system state in order to change the probability amplitudes of the corresponding states. Thus, the quantum strategies are represented by unitary operators on a Hilbert space (3) that the users apply locally to their qubit on the entangled state and transform the whole system behavior accordingly.

$$U_i(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} & e^{i\phi}\sin\frac{\theta}{2} \\ -e^{-i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{3}$$

The selection of some 0< < and 0< <2 implies covering linear combinations of strategies whose application drops outcomes that are not different from the classic game with mixed strategies and other combinations of (,) which lead to outcomes that are not possible in the classic game and consequently new equilibrium points emerge [16].

$$\left|H(0)\right|^2 = \left|H(1)\right|^2 = \left|H(2)\right|^2 = \begin{bmatrix} 1 & h & h \\ h & 1 & h \\ h & h & 1 \end{bmatrix} \tag{4}$$

In order to clarify some concepts we present the case of N=3 players transmitting over the same number of channels. Besides, for the sake of simplicity and without affecting the problem generality, we consider the normalization   hij (k) hen i=j and   hij (k)=h for any other case, being (4) the network matrix. Suppose A, B and C denote three players whose utilities are calculated using expression (1), that is, the classical case. For instance, the player A utilities are displayed in Table 1 as function of his and the other users actions. As the table shows, the highest utility results when he defects and the other players cooperate   ABC =7.65. On the other hand, if the other players betray while A cooperates, he receives a significantly smaller payoff. Because of that, classically all players decide to betray on average and resulting 2.42 the channel capacity for any user. In other words, if, for example, it is supposed the minimum Capacity Cj admitted is 2, the players will prefer a clear communication at the expense of the battery drain. This clearly is the Nash Equilibrium for the network and the corresponding payoff results less than the one the players would achieve if they all cooperate. This situation is shown in Figure 1 where the user C payoff
is depicted as function of others users actions. The upper surface corresponds to the case of user C deciding to Betray and the lower surface graph arises from the C decision to Cooperate.

Table 1: Player A capacity for SNR=100 and h=0.23. 0=Cooperate and 1=Defect. The highest utility results when A defects and the others cooperate. On the other hand, if the others betray while A cooperates, the last receive significantly smaller payoff. Because of the symmetry, the same occurs with other players payoffs.

$\|ABC\rangle$	C_A
$\|000\rangle$	3.3291
$\|001\rangle$	1.8339
$\|010\rangle$	1.8339
$\|011\rangle$	1.425
$\|100\rangle$	7.65
$\|101\rangle$	3.44
$\|110\rangle$	3.44
$\|111\rangle$	2.42

When the problem is analyzed from the point of view of quantum computation, that is, when the initial state is entangled and the strategy space is spanned to add new strategies, it is possible to present new favorable conditions to the users. In other words, they can make use of some strategy which leads to a more favorable situation for the entire network.

Preparing the system in the entangled state (2) defined above; the players choose their strategies according to their preferences and their previous experience. Meanwhile, the classic strategy "Cooperate" is represented by U(0,0) while betraying is represented by strategy U(2), and, as **Figure 2** shows, the Player C payoff depends on the (,) combinations.

The data of C payoff depicted in the figure arise when the strategies of A and B are Q=U(,0). As a consequence, it is observed that player C best strategy is also to choose U(,0) and because of the problem symmetry, Q results the best strategy for every player. Moreover, this is a Pareto optimum due to none user is willing to change because the payoff would be less than 3.3291, the one corresponding to the strategy, 000 .

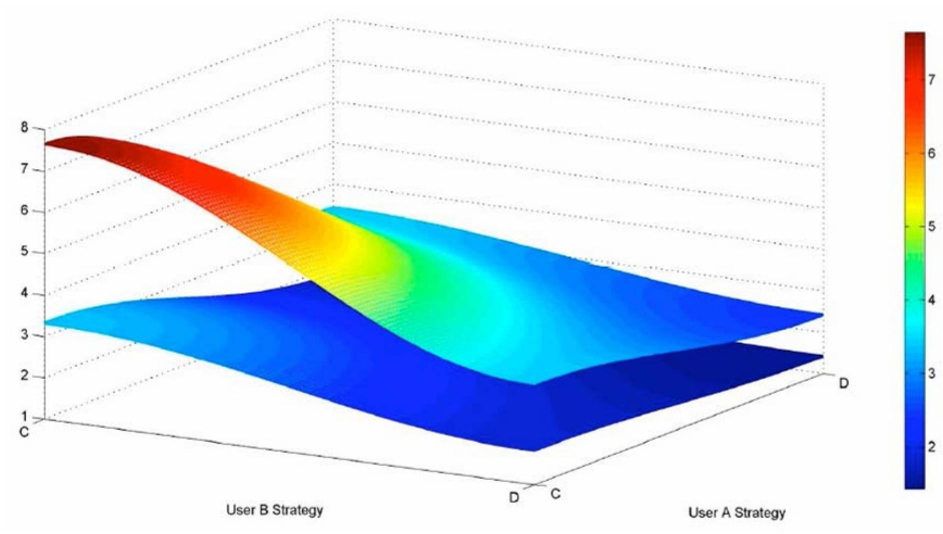

Figure 1: Player's C payoff as function of clearly (and . Users A and B play both strategy Q =U (π,0) and then, the best strategy for C is to play also U(,0) ,0) The maximum payoff is

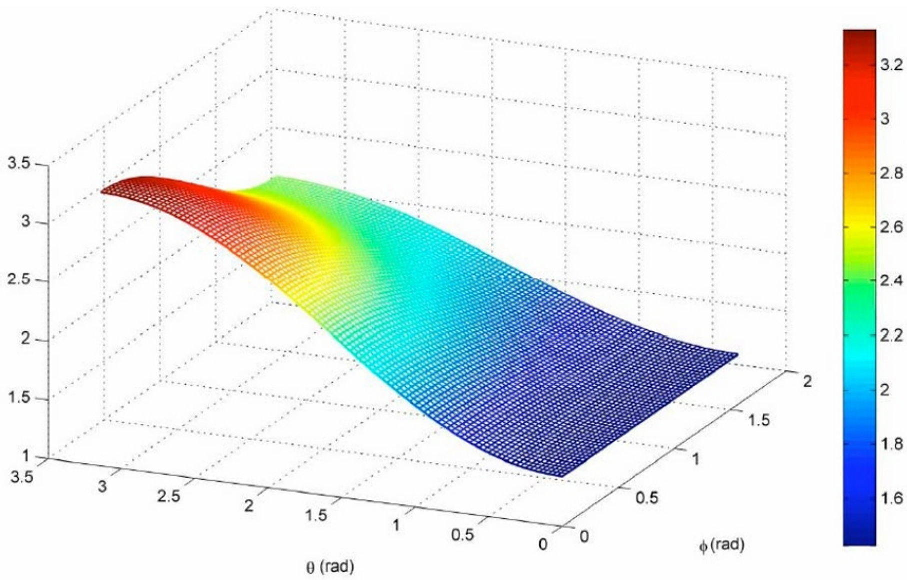

Figure 2: Player C payoff as function of (and Users A and B play both strategy Q =U′(π,0) and then, the best strategy for C is to play also U(′(π,0) .).

Consequently, the quantum model offers the users a different way of stable cooperation, allowing better transmissions and less battery drain.

3. CONCLUSIONS

We have proposed a novel quantum game application. The selfish behavior of users in a wireless network can be naturally modeled by a game. The nodes of the network are the players and the payoffs are represented by the users transmission rate. The classic strategies are Cooperate with the

whole network, which implies to direct all the power to one channel, or to betray, distributing the power to all the
channels causing interference to other users and diminishing their SINR. The quantum game of interference can describe classic users behavior but also permits to design new power control techniques for improving the actual ones. Cooperating is the best choice because of power saving and interference avoiding but is not an equilibrium condition, since any player can do better if change the strategy to "Betray". On the other hand, the use of quantum entanglement make possible the use of a different strategy which drives to a Pareto optimal equilibrium, guaranteeing the best possible performance for the whole network in the sense of better transmission rate and more power saving.

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