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Aims and Scope

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Contents

AN ALGORITHM FOR SOLVING A CAPACITATED FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC RANSPORTATION PROBLEM

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A B S T R A C T

Abstract: In this paper, a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. An algorithm to find the efficient cost-time trade off pairs in a capacitated fixed charge bicriterion indefinite quadratic transportation problem is developed. The algorithm is based on the concept of solving the indefinite quadratic fixed charge transportation problem and reading the corresponding time from the time matrix. It is illustrated with the help of a numerical example.

Keywords: optimum time cost trade off , capacitated transportation problem, fixed charge, bi-criterion indefinite quadratic transportation problem.

1. Introduction

In the classical transportation problem, the cost of transportation is directly proportional to the number of units of the commodity transported. But in real world situations, when a commodity is transported , a fixed cost is incurred in the objective function .The fixed cost may represent the cost of renting a vehicle, landing fees at an airport, set up cost for machines etc. The fixed charge transportation problem was originally formulated by G.B Dantzig and W. Hirisch [13] in 1954. After that , several procedures for solving fixed charge transportation problems were developed. Sometimes, there may exist emergency situations such as fire services, ambulance services, police services etc when the time of transportation is more important than cost of transportation. Hammer[12], Szwarc[14], Garfinkel et al. [7], Bhatia et al. [5] and many others have studied the time minimizing transportation problem which is a special case of bottleneck linear programming problems.Another important class of transportation problem consists of capacitated transportation problem. Many researchers like Gupta et.al. [8-11], Dahiya et.al. [6] have contributed in this field. In 1976, Bhatia et .al. [4] provided the time cost trade off pairs in a linear transportation problem. Then in 1994, Basu et.al. [3] developed an algorithm for finding the optimum time cost trade off pairs in a fixed charge linear transportation problem giving same priority to cost and time.

Another class of transportation problems , where the objective function to be optimized is the product of two linear functions which gives more insight in to the situation than the optimization of each criterion. Arora et. al. [1-2] have contributed in the field of indefinite quadratic transportation problem. In this paper, a capacitated fixed charge indefinite quadratic transportation problem giving same priority to cost and time is studied. An algorithm to identify the efficient cost time trade off pairs for the problem is developed.

2. MATHEMATICAL MODEL FOR A CAPACITATED FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM:

$$\text{(P1):}\ \min\left\{\left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)+\sum_{i\in I} F_i, \max_{i\in I, j\in J}\left(t_{ij} / x_{ij}>0\right)\right\}$$

subject to

$$\sum_{j\in J} x_{ij} \le a_i; \forall i \in I \qquad (1)$$

$$\sum_{i\in I} x_{ij} = b_j; \forall j \in J \qquad (2)$$

$$l_{ij} \le x_{ij} \le u_{ij}\ ; \forall (i, j) \in I \times J \qquad (3)$$

$I = \{1, 2, \dots m\}$ is the index set of m origins.

$J = \{1, 2, \dots, n\}$ is the index set of n destinations.

x_{ij} = number of units transported from origin i to the destination j.

c_{ij} = variable cost of transporting one unit of commodity from i^{th} origin to the j^{th} destination.

d_{ij} = the per unit damage cost or depreciation cost of commodity transported from i^{th} origin to the j^{th} destination.

l_{ij} and u_{ij} are the bounds on number of units to be transported from i^{th} origin to j^{th} destination.

t_{ij} is the time of transporting goods from ith origin to the j^{th} destination.

F_i is the fixed cost associated with i^{th} origin. The fixed cost F_i depends upon the amount supplied from the i^{th} origin to different destinations and is defined as follows.

$$F_i = \sum_{l=1}^{p} F_{il}\delta_{il} \quad , i=1, 2, 3\dots m \quad , l=1,2,3\dots p$$

$$\text{where } \delta_{il} = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} x_{ij} > a_{il} \\ 0 & \text{otherwise} \end{cases} \text{ for } l=1,2,3\ldots\ldots.p,\ i=1,2,\ldots\ldots m$$

Here, $0 = a_{i1} < a_{i2} \ldots < a_{ip}$ .Also a_{i1} , $a_{i2} \ldots$, a_{ip} ($i = 1,2, \ldots m$) are constants and F_{il} are the fixed costs $i= 1, 2 \ldots m, l = 1,2 ..p$

In the problem (P1), we need to minimize the total transportation cost and depreciation cost simultaneously to be transported from the i^{th} origin to the j^{th} destination Also we need to find the different cost - time trade off pairs.

3. THEORETICAL DEVELOPMENT:

Since $\sum_{i \in I} a_i > \sum_{i \in J} b_j$, the given problem is not a balanced problem . Therefore we introduce a dummy destination (n+1) with demand $b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ .The cost and time allocated in this $(n+1)^{th}$ column is zero. The resulting balanced problem is given below.

$$(P1'): \min \left\{ \left(\sum_{i \in I} \sum_{j \in J'} c_{ij} x_{ij} \right) \left(\sum_{i \in I} \sum_{j \in J'} d_{ij} x_{ij} \right) + \sum_{i \in I} F_i, \max_{i \in I, j \in J'} \left(t_{ij} / x_{ij} > 0 \right) \right\}$$

subject to

$$\sum_{j=1}^{n+1} x_{ij} = a_i; \forall i \in I \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} = b_j; \forall j \in J' \tag{5}$$

$$l_{ij} \le x_{ij} \le u_{ij}; \forall i \in I, j \in J \tag{6}$$

where $J' = \{1,2,3\ldots\ldots n,n+1\}$ and $x_{i,n+1} \ge 0; l_{i,n+1} = 0; u_{i,n+1} \ge 0; \forall i \in I$

$$c_{i,n+1} = 0 = d_{i,n+1} = t_{i,n+1}; \forall i \in I$$

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

F_i for i=1,2,…m are defined as in problem (P1).

In order to solve the problem (P1´), we separate it in to two problems (P2) and (P3) where

(P2): minimize the cost function $\left\{\left(\sum_{i\in I}\sum_{j\in J'} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J'} d_{ij}x_{ij}\right)+\sum_{i\in I}F_i\right\}$ subject to (4), (5) and (6) .

(P3): minimize the time function $\left\{\max_{i\in I, j\in J'}\left(t_{ij} / x_{ij} > 0\right)\right\}$ subject to (4), (5) and (6).

To obtain the set of efficient cost- time trade off pairs, we first solve the problem (P2) and read the time with respect to the minimum cost Z where time T is given by the problem (P3) .

At the first iteration , let Z_1 * be the minimum total cost of the problem (P2).Find all alternate solutions i.e. solutions having the same value of $Z = Z_1$*. Let these solutions be $X_1, X_2, \ldots X_n$. Corresponding to these solutions, find the time T_1* =

$$\min_{X_1, X_2 \ldots X_n} \left\{\max_{i\in I, j\in J'}\left(t_{ij} / x_{ij} > 0\right)\right\}.$$

Then (Z_1* , T_1*) is called the first cost time trade off pair.Modify the cost with respect to the

time so obtained i.e. define $c_{ij} = \begin{cases} M & \text{if } t_{ij} \geq T^* \\ c_{ij} & \text{if } t_{ij} < T^* \end{cases}$ and form the new problem (P2´) and find its

optimal solution and all feasible alternate solutions. Let the new value of Z be Z_2* and the corresponding time is T_2* , then (Z_2*, T_2*) is the second cost time trade off pair. Repeat this process. Suppose that after q th iteration ,the problem becomes infeasible. Thus, we get the following complete set of cost- time trade off pairs. (Z_1*, T_1*) , (Z_2*, T_2*),(Z_3*,T_3*),…………….(Z_q*, T_q*) where e $Z_1^* \leq Z_2^* \leq Z_3^* \leq \ldots \leq Z_q^*$ and $T_1^* > T_2^* > T_3^* \ldots > T_q^*$. The pairs so obtained are pareto optimal solution of the given problem. Then we identify the minimum cost Z_1* and minimum time T_q* among the above trade off pairs. The pair (Z_1*, T_q*) with minimum cost and minimum time is termed as the ideal pair which can not be achieved in practical situations.

Theorem 1: Let X = {X_{ij}} be a basic feasible solution of problem (P2) with basis matrix B. Then it will be an optimal basic feasible solution if

$$R_{ij}^{1} = \theta_{ij}\left[z_1(d_{ij}-z_{ij}^{2})+z_2(c_{ij}-z_{ij}^{1})+\theta_{ij}(c_{ij}-z_{ij}^{1})(d_{ij}-z_{ij}^{2})\right] + \Delta F_{ij} \geq 0; \forall (i,j) \in N_1$$

and

$$R_{ij}^{2} = \theta_{ij}\left[\theta_{ij}(c_{ij}-z_{ij}^{1})(d_{ij}-z_{ij}^{2})-z_1(d_{ij}-z_{ij}^{2})-z_2(c_{ij}-z_{ij}^{1})\right] + \Delta F_{ij} \geq 0; \forall (i,j) \in N_2$$

such that

$$u_i^1 + v_j^1 = c_{ij} \quad \forall (i,j) \in B \tag{7}$$

$$u_i^2 + v_j^2 = d_{ij} \quad \forall (i,j) \in B \tag{8}$$

$$u_i^1 + v_j^1 = z_{ij}^1 \quad \forall (i,j) \in N_1 \text{ and } N_2 \tag{9}$$

$$u_i^2 + v_j^2 = z_{ij}^2 \quad \forall (i,j) \in N_1 \text{ and } N_2 \tag{10}$$

ΔF_{ij} is the change in fixed cost $\sum_{i\in I} F_i$ when some non basic variable x_{ij} undergoes change by an amount of θ_{ij}.

Z1 = value of $\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

Z_2 = value of $\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

θ_{ij} = level at which a non basic cell (i,j) enters the basis replacing some basic cell of B.N1 and N2 denotes the set of non basic cells (i,j) which are at their lower bounds and upper bounds respectively.

Note: $u_i^1, v_j^1, u_i^2, v_i^2$ are the dual variables which are determined by using equations (7) to (10) and taking one of the u_i,'s or v_j,'s.

Proof: Let Z^0 be the objective function value of the problem (P2).

Let $z^0 = Z_1Z_2 + F^0$ where $F^0 = \sum_{i\in I} F_i$

Let $\hat{z}$ be the objective function value at the current basic feasible solution $\hat{X} = \{x_{ij}\}$ corresponding to the basis B obtained on entering the non basic cell $x_{ij} \in N_1$ in to the basis which undergoes change by an amount θ_{ij} and is given by min $\{u_{ij} - l_{ij}$; $x_{ij} - l_{ij}$ for all basic cells (i,j) with a ($-\theta$) entry in the θ-loop; $u_{ij} - x_{ij}$ for all basic cells (i,j) with a ($+\theta$) entry in the θ-loop$\}$.

Then $\hat{z} = \left[z_1 + \theta_{ij}\left(c_{ij} - z_{ij}^1\right)\right]\left[z_2 + \theta_{ij}\left(d_{ij} - z_{ij}^2\right)\right] + F^0 + \Delta F_{ij}$

$$\hat{z} - z^0 = \left[z_1 z_2 + \theta_{ij} z_1 (d_{ij} - z^2_{ij}) + z_2 \theta_{ij} (c_{ij} - z^1_{ij}) + \theta^2_{ij} (c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) - z_1 z_2\right] + \Delta F_{ij}$$

$$= \theta_{ij}\left[z_1 (d_{ij} - z^2_{ij}) + z_2 (c_{ij} - z^1_{ij}) + \theta_{ij} (c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij})\right] + \Delta F_{ij}$$

This basic feasible solution will give an improved value of z if $\hat{z} < z^0$ .It means

$$\text{If } \theta_{ij}\left[z_1 (d_{ij} - z^2_{ij}) + z_2 (c_{ij} - z^1_{ij}) + \theta_{ij} (c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij})\right] + \Delta F_{ij} < 0 \qquad (11)$$

Therefore one can move from one basic feasible solution to another basic feasible solution on entering the cell (i,j) $\in$ N1 in to the basis for which condition (11) is satisfied.

It will be an optimal basic feasible solution if

$$R^1_{ij} = \theta_{ij}\left[z_1 (d_{ij} - z^2_{ij}) + z_2 (c_{ij} - z^1_{ij}) + \theta_{ij} (c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij})\right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_1$$

Similarly, when non basic variable xij $\in$N2 undergoes change by an amount θ_{ij} then

$$\hat{z} - z^0 = \theta_{ij}\left[\theta_{ij} (c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) - z_1 (d_{ij} - z^2_{ij}) - z_2 (c_{ij} - z^1_{ij})\right] + \Delta F_{ij} < 0$$

It will be an optimal basic feasible solution if

$$R^2_{ij} = \theta_{ij}\left[\theta_{ij} (c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) - z_1 (d_{ij} - z^2_{ij}) - z_2 (c_{ij} - z^1_{ij})\right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_2$$

4. ALGORITHM:

Step1: Given a capacitated fixed charge bi-criterion indefinite quadratic transportation problem (P1). Introduce a dummy destination to form the related balanced transportation problem (P1′) and then separate it in to two problems (P2) and (P3). Find a basic feasiblesolution of problem (P2) with respect to variable cost only. Let B be its corresponding basis.

Step 2: Calculate the fixed cost of the current basic feasible solution and denote it by F(current)

where $F(\text{current}) = \sum_{i=1}^{m} F_i$

Step 3(a): Find ΔF_{ij}= F(NB) -F (current) ij where F(NB) is the total fixed cost obtained when some non basic cell (i,j) undergoes change.

Step 3(b): Calculate θ_{ij} ,$(c_{ij}-z^1_{ij})$, $(d_{ij} - z^2_{ij})$, z_1, z_2

$$u^1_i + v^1_j = c_{ij} \qquad \forall (i, j) \in B$$

$$u^2_i + v^2_j = d_{ij} \qquad \forall (i, j) \in B$$

$$u^1_i + v^1_j = z^1_{ij} \qquad \forall (i, j) \in N_1 \text{ and } N_2$$

$$u_i^2 + v_j^2 = z_{ij}^2 \quad \forall (i,j) \in N_1 \text{ and } N_2$$

Z_1 = value of $\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

Z_2 = value of $\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

θ_{ij} = level at which a non basic cell (i,j) enters the basis replacing some basic cell of B.N_1 and N_2 denotes the set of non basic cells (i,j) which are at their lower bounds and upper bounds respectively.

Note: $u_i^1, v_j^1, u_i^2, v_j^2$ are the dual variables which are determined by using above equations and taking one of the u_i,s or v_j,s as zero.

Step 3(c) : Find $R_{ij}^1 \forall (i,j) \in N_1$ and $R_{ij}^2 \forall (i,j) \in N_2$ where

$$R_{ij}^1 = \theta_{ij}\left[z_1(d_{ij}-z_{ij}^2)+z_2(c_{ij}-z_{ij}^1)+\theta_{ij}(c_{ij}-z_{ij}^1)(d_{ij}-z_{ij}^2)\right] + \Delta F_{ij} \geq 0; \forall (i,j) \in N_1 \text{ and}$$

$$R_{ij}^2 = \theta_{ij}\left[\theta_{ij}(c_{ij}-z_{ij}^1)(d_{ij}-z_{ij}^2)-z_1(d_{ij}-z_{ij}^2)-z_2(c_{ij}-z_{ij}^1)\right] + \Delta F_{ij} \geq 0; \forall (i,j) \in N_2$$

Step 4: If $R_{ij}^1 \geq 0 \forall (i,j) \in N_1$ and $R_{ij}^2 \geq 0 \forall (i,j) \in N_2$ then the current solution so obtained is the optimal solution to (P2).Go to step 5.Otherwise, some (i,j)∈N_1 for which $R_{ij}^1 < 0$ or some (i,j)∈N_2 for which $R_{ij}^1 < 0$ will undergo change. Go to step 2.

Step 5: Let Z_1 be the optimal cost of (P2′) yielded by the basic feasible solution $\{y'_{ij}\}$. Find all alternate solutions to the problem (P2) with the same value of the objective function.

Let these so lutions be $X_1, X_2, \ldots, X_n$ and $T_1 = \min_{X_1, X_2 \ldots X_n}\left\{\max_{i\in I, j\in J'}\left(t_{ij} / x_{ij} > 0\right)\right\}$.

Then the corresponding pair (Z_1 , T_1) will be the first time cost trade off pair for the problem (P1).To find the second cost- time trade off pair, go to step 6.

Step6: Define $c_{ij}^1 = \begin{cases} M & \text{if } t_{ij} \geq T^1 \\ c_{ij} & \text{if } t_{ij} < T^1 \end{cases}$

where M is a sufficiently large positive number. Form the corresponding capacitated fixed charge quadratic transportation problem with variable cost c_{ij}^1. Repeat the above process till the problem becomes infeasible. The complete set of time cost trade off pairs of (P1) at the end of q^{th} iteration are given by $(Z_1, T_1), (Z_2, T_2)$………(Z^q, T^q) where $Z^1 \leq Z^2 \leq \ldots \leq Z^q$ and $T^1 > T^2 > \ldots > T^q$.

Remark 2: The pair (Z^1, T^q) with minimum cost and minimum time is the ideal pair which can not be achieved in practice except in some trivial case.

Convergence of the algorithm: The algorithm will converge after a finite number of steps because we are moving from one extreme point to another extreme point and the problem becomes infeasible after a finite number of steps.

5. NUMERICAL ILLUSTRATION:

Consider a 3 x 3 capacitated fixed charge bi-criterion indefinite quadratic transportation problem. Table 1 gives the values of c_{ij}, d_{ij}, a_i, b_j for i=1,2,3 and j=1,2,3 Table 1:Cost matrix of problem (P1)

	D_1	D_2	D_3	a_i
O_1	5 4	9 2	9 1	30
O_2	4 3	6 7	2 4	40
O_3	2 2	1 9	1 4	80
b_j	30	20	30	

Note: values in the upper left corners are c_{ij} 's and values in lower left corners are d_{ij} 's for i=1,2,3.and j=1,2,3.

$1 \leq x_{11} \leq 10$, $2 \leq x_{12} \leq 10$, $0 \leq x_{13} \leq 5$,$0 \leq x_{21} \leq 15$, $3 \leq x_{22} \leq 15$, $1 \leq x_{23} \leq 20$,$0 \leq x_{31} \leq 20$, $0 \leq x_{32} \leq 13$, $0 \leq x_{33} \leq 25$

$F_{11} = 100$, $F_{12} = 50$, $F_{13} = 50$, $F_{21} = 150$, $F_{22} = 100$, $F_{23} = 50$,$F_{31} = 200$, $F_{32} = 150$, $F_{33} = 100$

$F_i = \sum_{l=1}^{3} F_{il}\delta_{il}$ for l=1,2,3 where for i= 1,2,3

$$\delta_{i1} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i2} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i3} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 20 \\ 0 & \text{otherwise} \end{cases}$$

Table 2 gives the values of t_{ij} ,s for i=1,2,3 and j=1,2,3

Table 2 :values of t_{ij}

	D_1	D_2	D_3
O_1	15	8	13
O_2	10	13	11
O_3	12	10	9

Introduce a dummy destination in Table 1 with $c_{i4} = 0 = d_{i4}$ for all i = 1,2 ,3 .and b_4 =70 Now we find an initial basic feasible solution of problem (P2) which is given in table 3 below.

Table 3: A basic feasible solution of problem (P2)

	D_1	D_2	D_3	D_4	u_i^1	u_i^2	F(current)
O_1	5 $\underline{1}$ 4	9 $\underline{2}$ 2	9 $\underline{0}$ 1	0 **27** 0	0	0	100
O_2	4 **9** 3	6 **5** 7	2 **5** 4	0 **21** 0	0	0	250
O_3	2 2 $\overline{20}$	1 9 $\overline{13}$	1 4 $\overline{25}$	0 **22** 0	0	0	450
v_j^1	4	6	2	0			
v_j^2	3	7	4	0			

Note: entries of the form $\underline{a}$ and $\overline{b}$ represent non basic cells which are at their lower and upper bounds respectively. Entries in bold are basic cells.

F(current) = 800, $z_1 = 177$, $z_2 = 347$

Table 4: Calculation of optimality condition

NB	O_1D_1	O_1D_2	O_1D_3	O_3D_1	O_3D_2	O_3D_3
θ_{ij}	9	2	4	6	10	15
$c_{ij} - z_{ij}^1$	1	3	7	-2	-5	-1
$d_{ij} - z_{ij}^2$	1	-5	-3	-1	2	0
F(NB)	750	800	800	850	850	850
ΔF_{ij}	-50	0	0	50	50	50
R_{ij}^1	4747	252	7256			
R_{ij}^2				5348	12860	5255

Since $R_{ij}^1 \geq 0 \;\; \forall (i,j) \in N_1$ and $R_{ij}^2 \geq 0; \forall (i,j) \in N_2$, the solution in table 3 is an optimal solution of (P2). Therefore minimum cost = Z^1 = (177 x 347) + 800 = 62219 and corresponding time is $T^1 = 15$. Hence the first time cost trade off pair is (62219,15)

$$\text{Define } c_{ij}^1 = \begin{cases} M & \text{if } t_{ij} \geq T^1 = 15 \\ c_{ij} & \text{if } t_{ij} < T^1 = 15 \end{cases}$$

rade off pair as (62219,13). Similarly third cost - time trade off pair is (62471, 12). After that the problem becomes infeasible. Hence the cost time trade off pairs are (62219, 15) , (62219,13),(62471,12).

6. CONCLUSION

In order to solve a capacitated fixed charge bi – criterion indefinite quadratic transportation problem , we separate the problem in to two problems. One of them being an indefinite quadratic programming problem has its optimal solution at its extreme point. After finding the optimal solution we read the time from the time matrix corresponding to $x_{ij} > 0$. Then we define the new cost and new problem with these costs to find the second cost - time trade off pair. The process is repeated till the problem becomes infeasible. The process must end after a finite number of steps because our algorithm moves from one extreme point to another which is finite in number.

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ON THE STRENGTH AND WEAKNESS OF BINOMIAL MODEL FOR PRICING VANILLA OPTIONS

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A B S T R A C T

Abstract: This paper presents binomial model for pricing vanilla options. Binomial model can be used to accurately price American style options than the Black-Scholes model as it takes into consideration the possibilities of early exercise and other factors like dividends. The strength and weakness of this model were considered. This model is both computationally efficient and accurate but not adequate to deal with path dependent options.

Keywords: American Option, Binomial Model, Black Scholes Model, European Option, Vanilla option.

1. Introduction

In the past two decades, options have undergone a transformation from specialized and obscure securities to ubiquitous components of the portfolio of not only large fund managers, but also ordinary investors. Essential ingredients of any successful modern investment strategy include the ability to generate income streams and reduce risk, as well as some level of speculation all of which can be accomplished by effective use of options.An option is a financial contract or a contingent claim that gives the holder the right, but not the obligation to buy or sell an underlying asset for a predetermined price called the strike or exercise price during a certain period of time. Options come in a variety of “flavours”. A vanilla option offers the right to buy or sell an underlying security by a certain date at a set strike price. In comparison to other option structures, vanilla options are not fancy or complicated. Such options may be well-known in the markets and easy to trade. Increasingly, however, the term vanilla option is a relative measure of complexity, especially when investors are considering various options and structures. Examples of vanilla options are an American option which allows exercise at any point during the life of the option and a European option that allows exercise to occur only at expiration.Black and Scholes published their seminar work on option pricing [1] in which they described a mathematical frame work for finding the fair price of a European option. They used a no-arbitrage argument to describe a partial differential equation which governs the evolution of the option price with respect to the maturity time and the price of the underlying asset.

The subject of numerical methods in the area of option pricing and hedging is very broad, putting more demands on computation speed and efficiency. A wide range of different types of contracts are available and in many cases there are several candidate models for the stochastic evolution of the underlying state variables [10].Now, we present an overview of binomial model in the context of Black-Scholes-Merton [1, 8] for pricing vanilla options based on a risk-neutral valuation which was first suggested and derived by Cox-Ross-Rubinstein [4] and assumes that stock price movements are composed of a large number of small binomial movements. Other procedures are finite difference methods for pricing derivative governed by solving the underlying partial differential equations was considered by Brennan and Schwarz [3] and Monte Carlo method for pricing European option and path dependent options was introduced by Boyle [2]. The comparative study of finite difference method and Monte Carlo method for pricing European option was considered by Fadugba, Nwozo and Babalola [6]. Later, on the stability and accuracy of finite difference method for option pricing was considered by Fadugba and et al [5]. These procedures provide much of the infrastructure in which many contributions to the field over the past three decades have been centered.In this paper we shall consider only the strength and weakness of binomial model for pricing vanilla options namely American and European options.

2.0 BINOMIAL MODEL

This is defined as an iterative solution that models the price evolution over the whole option validity period. For some vanilla options such as American option, iterative model is the only choice since there is no known closed form solution that predicts its price over a period of time. The Cox-Ross-Rubinstein "Binomial" model [4] contains the Black-Scholes analytic formula as the limiting case as the number of steps tends to infinity. Next we shall present the derivation and the implementation of the binomial model below.

2.1 THE COX-ROSS-RUBINSTEIN MODEL [4, 7]

We know that after a period of time, the stock price can move up to *Su* with probability *p* or down to *Sd* with probability $(1-\text{p})$, where $u > 1$ and $0 < d < 1$. Therefore the

$$f_u = \max(Su - K, 0) \quad (1)$$

$$f_d = \max(S_d - K, 0) \quad (2)$$

Where *fu* and *fd* are the values of the call option after upward and downward movements respectively.

We need to derive a formula to calculate the fair price of vanilla options. The risk neutral call option price at the present time is given by

$$f = e^{-r\delta t}[pf_u + (1-p)f_d] \tag{3}$$

Where the risk neutral probability is given by

$$p = \frac{e^{r\delta t} - d}{u - d} \tag{4}$$

Now, we extend the binomial model to two periods. Let *fuu* denote the call value at time 2δt for two consecutive upward stock movements, *f*ud for one downward and one upward movement and *fdd* for two consecutive downward movements of the stock price [9]. Then we have

$$f_{uu} = \max(Suu - K, 0)$$

(5)

$$f_{ud} = \max(Sud - K, 0)$$

(6)

$$f_{dd} = \max(Sdd - K, 0)$$

(7)

The values of the call options at time δt are

$$f_u = e^{-r\delta t}[pf_{uu} + (1-p)f_{ud}] \tag{8}$$

$$f_d = e^{-r\delta t}[pf_{ud} + (1-p)f_{dd}] \tag{9}$$

Substituting (8) and (9) into (3), we have

$$f = e^{-r\delta t}[pe^{-r\delta t} f_{uu} + (1-p)f_{ud} + (1-p)e^{-r\delta t}(pf_{ud} + (1-p)f_{dd})]$$

$$f = e^{-2r\delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})] \tag{10}$$

Equation (10) is called the current call value, where the numbers p^2, $2p(1-p)$ and $(1-p)^2$ are the risk neutral probabilities for the underlying asset prices *Suu* , *Sud* and *Sdd* respectively. We generalize the result in (10) to value an option at $T = N\delta t$ as follows

$$f = e^{-Nr\delta t} \sum_{j=0}^{N} {}^N C_j \, p^j (1-p)^{N-j} f_{u^j d^{N-j}}$$

$$f = e^{-Nr\delta t} \sum_{j=0}^{N} {}^N C_j \, p^j (1-p)^{N-j} \max(Su^j d^{N-j} - K, 0) \tag{11}$$

Where $f_{u^j d^{N-j}} = \max(Su^j d^{N-j} - K, 0)$ and ${}^N C_j = \dfrac{N!}{(N-j)!\,j!}$ is the binomial coefficient. We assume that m is the smallest integer for which the option's intrinsic value in (11) is greater than zero. This implies that $Su^m d^{n-m} \geq K$. Then (11) can be written as

$$f = Se^{-Nr\delta t} \sum_{j=0}^{N} {}^N C_j p^j (1-p)^{N-j} u^j d^{N-j}$$

$$- Ke^{-Nr\delta t} \sum_{j=0}^{N} {}^N C_j p^j (1-p)^{N-j} f_{u^j d^{N-j}} \tag{12}$$

Equation (12) gives us the present value of the call option.

The term $e^{-Nr\delta t}$ is the discounting factor that reduces f to its present value. We can see from the first term of (12) that -1is the binomial probability of j upward movements to occur after the first N trading periods and $Su^j d^{n-j}$ is the corresponding value of the asset after j upward movements of the stock price. The second term of (12) is the present value of the option's strike price. Let r t $Q = e^{-r\delta t}$, we substitute Q in the first term of (12) to yield

$$f = SQ^{-N} \sum_{j=0}^{N} {}^N C_j p^j (1-p)^{N-j} u^j d^{N-j}$$

$$- Ke^{-Nr\delta t} \sum_{j=0}^{N} {}^N C_j p^j (1-p)^{N-j} f_{u^j d^{N-j}}$$

$$f = S \sum_{j=0}^{N} {}^N C_j [Q^{-1} pu]^j [Q^{-1}(1-p)d]^{N-j}$$

$$- Ke^{-Nr\delta t} \sum_{j=0}^{N} {}^N C_j p^j (1-p)^{N-j} f_{u^j d^{N-j}} \tag{13}$$

Now, let $\Phi(m; N, p)$ be the binomial distribution function given by

$$\Phi(m; N, p) = \sum_{j=0}^{N} {}^N C_j p^j (1-p)^{N-j} \tag{14}$$

Equation (14) is the probability of at least m success in N independent trials, each resulting in a success with probability p and in a failure with probability y $(1-p)$. Then let $p' = Q^{-1} pu$ and $(1-p') = Q^{-1}(1-p)d$

Consequently, it follows that

$$f = S\Phi(m; N, p') - Ke^{-rT}\Phi(m; N, p) \tag{15}$$

The model in (15) was developed by Cox-Ross-Rubinstein [6], where $\delta t = \frac{T}{N}$ and we will refer to it as CRR model. The corresponding put value of the European option can be obtained using call put relationship of the form $C_E + Ke^{-rt} = P_E + S$ as

$$f = Ke^{-rT}\Phi(m; N, p) - S\Phi(m; N, p') \quad (16)$$

Where the risk free interest rate is denoted by r , C_E is the European call, P_E is the European put and S is the initial stock price. European option can only be exercised at expiration, while for an American option, we check at each node to see whether early exercise is advisable to holding the option for a further time period δt . When early exercise is taken into consideration, the fair price must be compared with the option's intrinsic value [7].

2.2 NUMERICAL IMPLEMENTATION

Now, we present the implementation of binomial model for pricing vanilla options as follows.

When stock price movements are governed by a multi-step binomial tree, we can treat each binomial step separately. The multi-step binomial tree can be used for the American and European style options.

Like the Black-Scholes, the CRR formula in (15) can only be used in the valuation of European style options and can easily be implemented in Matlab. To overcome this problem, we use a different multi-period binomial model for the American style options on both the dividend and non-dividend paying stocks. Now we present the Matlab implementation.

The stock price of the underlying asset for non-dividend and dividend paying stocks are given respectively by

$$Su^j d^{N-j}, j = 0,1,\ldots, N, N = 0,1,\ldots i-1 \quad (17)$$

$$S(1-\lambda)u^j d^{N-j}, j = 0,1,\ldots, N, N = i, i+1,\ldots \quad (18)$$

Where the dividend is denoted by λ that reduces underlying price of the asset.

For the European call and put options, the Matlab code takes into consideration on the prices at the maturity date T and the stock prices for non-dividend paying stocks in (17). The call and put prices of European option are given by (15) and (16) respectively.

For the American call and put options, the Matlab code will incorporate the early exercise privilege and the date T , when the dividend will be paid. Then, it implies that the stock prices will exhibit (17) and (18). The call and put prices of American option for non-dividend paying stock are given by

$$f = \max[S_T - K, (S\Phi(m; N, p') - Ke^{-rT}\Phi(m; N, p))] \quad (19)$$

$$f = \max[K - S_T, (Ke^{-rT}\Phi(m; N, p) - S\Phi(m; N, p'))]$$

(20)

For dividend paying stock, we replace (17) with (18) in (12) and substitute in (19) and (20) to get respectively the call and put prices of American option.

3.0 NUMERICAL EXAMPLES

Now, we present some numerical examples.

Example 1

We compute the values of vanilla options. The results in Tables 1 and 2 for both European and American options are compared to those obtained using Black-Scholes analytic pricing formula. The rate of convergence for binomial model may be assessed by repeatedly doubling the number of time step N . Tables 1 and 2 use the parameters below in computing the options prices as we increase the number of steps.

$$S = 45, K = 40, T = \frac{1}{2}, r = \frac{1}{10}, \sigma = \frac{1}{4}$$

The Black-Scholes price for call and put options are 7.6200 and 0.6692 respectively.Table 1: Comparison of the Binomial Model to Black-Scholes Value of the Option as we increase N

N	European call	American call	European put	American put
10	7.6184	7.6184	0.6676	0.7124
20	7.6305	7.6305	0.6797	0.7235
30	7.6042	7.6042	0.6534	0.7027
40	7.6241	7.6251	0.6742	0.7228
50	7.6070	7.6070	0.6562	0.7101
60	7.6219	7.6219	0.6710	0.7199
70	7.6209	7.6209	0.6701	0.7207
80	7.6124	7.6124	0.6616	0.7134
90	7.6210	7.6210	0.6702	0.7201
100	7.6216	7.6216	0.6707	0.7214

Table 2: The Comparison of the Convergence of the Binomial Model and Black-Scholes Value of Option as we double the value of N

N	European Call	American Call	European Put	American Put
20	7.6305	7.6305	0.6797	0.7235
40	7.6251	7.6251	0.6742	0.7228
60	7.6219	7.6219	0.6710	0.7199
80	7.6124	7.6124	0.6616	0.7134
100	7.6216	7.6216	0.6707	0.7214
120	7.6181	7.6181	0.6673	0.7182
140	7.6209	7.6209	0.6700	0.7211
160	7.6178	7.6178	0.6670	0.7184
180	7.6211	7.6211	0.6703	0.7213
200	7.6171	7.6171	0.6663	0.7185

Example 2

Consider pricing a vanilla option on a stock paying a known dividend yield with the following parameters:

$$S = 50, r = \frac{1}{10}, T = \frac{1}{2}, \sigma = \frac{1}{4}, \tau = \frac{1}{6}, \lambda = \frac{1}{20}$$

Table 3: Out of the Money, at the Money and in the Money Vanilla Options on a Stock Paying a Known Dividend Yield

K	European Call	American Call	Early Exercise Premium	European Put	American Put	Early Exercise Premium
30	18.97	20.50	1.53	0.004	0.004	0.00
45	6.06	6.47	0.41	1.37	1.49	0.12
50	3.32	3.42	0.10	3.38	3.78	0.40
55	1.62	1.63	0.01	6.40	7.31	0.91
70	0.11	0.11	0.00	19.19	21.35	2.16

4.0 DISCUSSION OF RESULTS

We can see from Table 1 that Black-Scholes formula for the European call option can be used to value its counterpart American call option for it is never optimal to exercise an American call option before expiration. As we increase the value of N , the value of the American put option is higher than the corresponding European put option as we can see from the above Tables because of the early exercise premium. Sometime the early exercise of the American put option can be optimal. Table 2 shows that binomial model converges faster and closer to the Black-Scholes value as the value of N is doubled. This method is very flexible in pricing vanilla option. Table 3 shows that the American option on the dividend paying stock is always worth more than its European counterpart. A very deep in the money, American option has a high early exercise premium. The premium of both put and call option decreases as the option goes out of the money. The American and European call options are not worth the same as it is optimal to exercise the American call early on a dividend paying stock. A deep out of the money, American and European call options are worth the same. This is due to the fact that they might not be exercised early as they are worthless. The above results can be obtained using Matlab codes.

5.0 CONCLUSION

Options come in many different flavours such as path dependent or non-path dependent, fixed exercise time or early exercise options and so on. Binomial model is suited to dealing with some of these option flavours.In general, binomial model has its strengths and weaknesses of use. This model is good for pricing options with early exercise opportunities, accurate, converges faster and it is relatively easy to implement but can be quite hard to adapt to more complex situations. We conclude that binomial model is good for pricing vanilla options most especially American and European options.

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BIANCHI TYPE-I COSMOLOGICAL MODELS WITH DUST FLUID IN GENERAL RELATIVITY

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A B S T R A C T

Abstract: In this paper, we have investigated Bianchi type -I cosmological model with dust fluid. The cosmological models are obtained by assuming the cosmological term inversely proportional to R (R is scale factor).Some physical properties of the cosmological models are also discussed.

Keywords:- Bianchi-I cosmological model Variable cosmological term. Dust fluid .

1. Introduction

The simplest homogeneous and anisotropic models are Bianchi type-I whose optical sections are that but the expansion or contraction rate is directional dependent. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding move general cosmological models than the isotropic Friedman- Robertson Walker models. The isotropy of the present day universe makes the Bianchi-I model a prime for studying the possible effects of an anisotropy in the early universe on modern day data observations.Solutions to the field equations may also be generated by law of variation of scale factor which was proposed by Pavon, D. (1991). The behavior of the cosmological scale factor R (t) in solution of Einstein's field equations with Robertson-Walker line elements has been the subject of numerous studies. In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Holy, F. et al(1997), Olson, T.S. et al. (1987), Beesham, A (1994), Maia, M.D. et al. (1994), Silveria, V. et al. (1994,1997), Torres, L.F.B. et al. (1996). Chen and Wu (1990) considered Λ varying R^{-2} (R is the scale factor) Carvalho and Lima (1992) generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background.

The idea of variable gravitational constant G in the framework of general relativity was first proposed by Dirac (1937). Lau (1983) working in the framework of general relativity, proposed modification linking the variation of G with that of Λ. . A number of authors investigated Bianchies models, using this approach (Abdel-Rahman 1990; Berman 1991; Kalligas et al. 1992; Abdussattar and Vishwakarma 1997; Vishwakarma 2005; Pradhan et al. 2006; Singh and Tiwari 2007). Borges and Carneiro (2005), Singh et al. (2007) have considered as cosmological term is proportional to the Hubble parameter in FRW model and Bianchi type-I model with variable G and Λ. In this paper we have investigated homogeneous Bianchi type -I space time with variables G and Λ containing matter in the form of dust fluid. We have obtained exact solutions of the field equations by assuming that cosmological term is inversely proportional to R.. The paper is organized as follows. Basic equations of the models in sec. 2. and solution in sec. 3. The physical behavior of the model is discussed in detail is last section.

2. METRIC AND FILED EQUATIONS :

We consider the space-time admitting Bianchi type-I group of motion in the form

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\, dy^2 + C^2(t)\, dz^2 \qquad .....(1)$$

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \qquad .....(2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

We take the equation of state

$$p = \omega\rho\ , \qquad 0 \leq \omega \leq 1 \qquad ....(3)$$

Here we take $\omega=0$, then $p=0$.

The Einstein's field equations with time dependent G and Λ given by (Weinberg 1972)

$$R_{ij} - ½\, R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda\, (t) g_{ij} \qquad .....(4)$$

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (4) yields.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda \qquad ....(5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G p + \Lambda \qquad ....(6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \qquad ....(7)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\rho+\Lambda \qquad ....(8)$$

The usual energy conservation equation $T^{j}_{i;j}=0,$ yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \qquad ....(10)$$

Equation (9) together with (10) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \qquad ....(11)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t. From (11) implying that Λ is a constant whenever G is constant. Using equation (3) in equation (10) and then integrating, we get,

$$\rho=\frac{k}{R^3} \qquad .....(12)$$

where k > 0 is constant of integration.

Let R be the average scale factor of Bianchi type -I universe i.e.

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{R^3} \qquad .....(14)$$

and $$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{R^3} \qquad .....(15)$$

where k_1 and k_2 are constant of integration. The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R}$$

$$\sigma=\frac{k}{\sqrt{3}R^3},$$

$$q=-1-\frac{\dot{H}}{H^2}=-\frac{-R\ddot{R}}{\dot{R}^2}$$

Equations (5)-(8) and (10) can be written in terms of H, σ and q as

$$H^2(2q-1)-\sigma^2 = 8\pi Gp - \Lambda \qquad .....(16)$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \qquad .....(17)$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0 \qquad .....(18)$$

From (16), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$ and $0 \le \frac{8\pi G\rho}{\theta^2} \le \frac{1}{3}$ for $\Lambda \ge 0$

Thus, the presence of positive Λ puts restriction on the upper limit of anisotropy, where as a negative Λ contributes to the anisotropy.

From (16), and (17), we have

$$\frac{d\theta}{dt} = -12\pi Gp - \frac{\theta^2}{2} + \frac{3\Lambda}{2} = 12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative Λ, and for positive Λ,

universe will slows down the rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implying that σ decreases in an evolving universe and it is negligible for infinitely large value of R.

3. SOLUTION OF THE FIELD EQUATIONS -

The system of equations (3), (5)-(8), and (11), supply only six equations in seven unknowns (A,B,C, ρ, p, G and Λ). One extra equation is needed to solve the system completely. The $\Lambda \alpha a^{-m}$ (a is scale factor and m is constant) considered by Olson, T.S. et al. (1987), Pavon, D. (1991), Maia, M.D. et al. (1994), Silveria, V. et. al. (1994, 1997) , Torres, LF. B. et al. (1996).

Thus we take the decaying vacuum energy density

$$\Lambda = \frac{c}{R} \qquad .....(19)$$

where c is positive constants. Using eq. (12) and (19) in eq. (11), we get

$$G = \frac{c}{16\pi k}\frac{R^2}{} \qquad .....(20)$$

From eq (16), (17), (19) and (20) we get

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{5c}{4R} = 0 \qquad .....(21)$$

Determining the time evolution of Hubble parameter, integrating (21), we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{c}{2}}\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ t + t_0\right]^{-1} \qquad . \qquad ....(22)$$

where t_o is a constant of integration, The integration constant is related to the choice of origin of time.

From eq. (22), we obtain the scale factor

$$R = \left(\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ \mathrm{t} + t_0\right)^2 \qquad ...(23)$$

By using eq (23) in (14) and (15), the metric (1), assumes the form

$$\begin{aligned} ds^2 = -dt^2 + \left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^4 \times \\ \Bigg[m_1^2 \ \exp 2\left\{\frac{(2k_1 + k_2)}{3} 2\sqrt{\frac{2}{c}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^{-5}\right\} dx^2 \\ + m_2^2 \ \exp 2\left\{\frac{(k_2 - k_1)}{3} 2\sqrt{\frac{2}{c}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^{-5}\right\} dy^2 \\ + m_3^2 \ \exp 2\left\{\frac{-(k_1 + 2k_2)}{3} 2\sqrt{\frac{2}{c}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^{-5}\right\} dz^2 \Bigg] \end{aligned} \qquad ....(24$$

where m_1, m_2 and m_3 are constants.

For the model (24), the spatial volume V, matter density ρ, pressure p gravitational constant G, cosmological constant Λ are given by

$$V = \left(\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ \mathrm{t} + t_0 \right)^6 \qquad .....(25)$$

$$\rho = \frac{k}{\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ t + \mathrm{to} \right]^6} \qquad .....(26)$$

p =0 ......(27)

$$G = \frac{c}{16\pi k}\left(\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ \mathrm{t} + t_0 \right)^4 \qquad .....(28)$$

$$\Lambda = c\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ t + t_0 \right]^{-2} \qquad ....(29)$$

Expansion scalar θ and shear σ are given by

$$\theta = 3\sqrt{\frac{c}{2}}\ \left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)} t + to \right]^{-1} \qquad ....(30)$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)} t + to \right]^{-6} \qquad .....(31)$$

The deceleration parameter q for the model is

$$q = -\frac{1}{2} \qquad ....(32)$$

Thus for the model (24), the spatial volume V is zero at t=t' where t' $= \dfrac{-t_0}{\frac{1}{2}\sqrt{\frac{c}{2}}}$ and expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at t=t' with an infinite rate of expansion. The scale factors also vanish at t=t' and hence the space-time exhibits point type singularity at initial epoch. The energy density, shear scalar diverges at the initial singularity. As t increases the scale factors and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also ρ, σ, ρ_v, ρ_c, Λ decrease as t increases. As $t \to \infty$ scale factors and volume become infinite whereas ρ, σ, ρ_v, ρ_c, and Λ tend to zero. Therefore, the model would essentially give an empty universe for large time t. Gravitational constant G(t) is zero at t =t' and as t increases G(t) also increases. The cosmological constant $\Lambda(t)\ \alpha\ 1/t^2$ which follows from the model of Kalligas et al. (1996); Berman (1990); Berman and Som (1990); Berman et al. (1989) . This form of Λ is physically reasonable as observations suggest that $\Lambda$is very small in the present universe.

The ratio $\frac{\sigma}{\theta} \to 0 \text{ as } t \to \infty$. So the model approaches isotropy for large value of t. Thus the

model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times.Finally, the solutions presented in the paper are new and useful or better understanding of the evolution of the universe in Bianchi type-I space-time with variable G and Λ.

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STIFF FLUID LRS BIANCHI -I COSMOLOGICAL MODELS WITH VARYING G AND Λ

Uttam Kumar Dwivedi*

A B S T R A C T

Abstract: I have discussed about a LRS Bianchi type-I cosmological model filled with stiff fluid, variable gravitational constant and cosmological constants. The cosmological models are obtained by assuming the cosmological term inverselyproportional to scale factor. The physical significance of the cosmological models are also discussed.

Key words:- LRS Bianchi type-I,Variable cosmological term. Stiff fluid

1. Introduction

After the cosmological constant was first introduced into general relativity by Einstein, its significance was studied by various cosmologists (for example Petrosian, V1975), but no satisfactory results of its meaning have been reported as yet. Zel'dovich (1968) has tried to visualize the meaning of this term from the theory of elementary particles. Further, Linde (1974) has argued that the cosmological term arises from spontaneous symmetry breaking and suggested that the term is not a constant but a function of temperature. It is also well known that there is a certain degree of anisotropy in the actual universe. Therefore, we have chosen the metric for the LRS cosmological model to be Bianchi type-I.

Solutions to the field equations may also be generated by law of variation of scale factor which was proposed by Pavon, D. (1991). In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Holy, F. et al(1997), Olson, T.S. et al. (1987), Pavon, D. (1991), Beesham, A (1994), Maia, M.D. et al. (1994), Silveria, V. et al. (1994,1997), Torres, L.F.B. et al. (1996).Chen and Wu (1990) considered Λ varying R^{-2} (R is the scale factor) Carvalho and Lima (1992) generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters . The idea of variable gravitational constant G in the framework of general relativity was first proposed by Dirac (1937). Lau (1983) working in the framework of general relativity, proposed modification linking the variation of G with that of Λ. This modification allows us to use Einstein's field equations formally unchanged since variation in Λ is accompanied by a variation of G. A number of authors investigated Bianchies models, using this approach (Abdel-Rahman 1990; Berman 1991; Sisterio1991; Kalligas et al. 1992; Abdussattar and Vishwakarma 1997; Vishwakarma 2000,2005; Pradhan et al. 2006; Singh et al 2007; Singh and Tiwari 2007 Tiwari, R.K 2008,).In this paper I have considered a LRS Bianchi type-I cosmological model with variables G and Λ filled with stiff fluid . We have obtained exact solutions of the field equations by assuming that cosmological term is inversely proportional to R (where R is scale factor). The paper is organized as follows. Basic equations of the models in sec. 2. and solution in sec. 3. The physical behavior of the model is discussed in detail is last section.

2. METRIC AND FILED EQUATIONS :

We consider spatially homogeneous and anisotropic LRS Bianchi type-I metric

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\,(dy^2 + dz^2) \quad .....(1)$$

The energy-momentum tensor T_{ij} for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \quad .....(2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

We take the equation of state (Zel'dovich 1962)

$$p = \rho \quad , \qquad \omega = 1 \quad .....(3)$$

The Einstein's field equations with time dependent G and Λ given by (Weinberg 1972)

$$R_{ij} - \tfrac{1}{2} R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda\,(t) g_{ij} \quad .....(4)$$

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (4) yields.

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 = -8\pi G p + \Lambda \quad ....(5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \quad ....(6)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 = 8\pi G \rho + \Lambda \quad ....(7)$$

In view of vanishing divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right)\right]+8\pi\rho\dot{G}+\dot{\Lambda}=0 \quad ....(8)$$

The usual energy conservation equation $T^{j}_{i;j}=0$, yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right)=0 \quad ....(9)$$

Equation (8) together with (9) puts G and Λ in some sort of coupled field given

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \quad ....(10)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t. From (10) implying that Λ is a constant whenever G is constant.

Let R be the average scale factor of LRS Bianchi type -I universe i.e.

$$R^3 = AB^2 \quad .....(11)$$

Using equation (3) in equation (9) and then integrating, we get,

$$. \quad \rho=\frac{k}{R^6} \quad .....(12)$$

where k > 0 is constant of integration.

From (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{R^3} \quad .....(13)$$

where k_1 is constant of integration. The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R}$$

$$\sigma=\frac{k}{\sqrt{3}R^3},$$

$$q=-1-\frac{\dot{H}}{H^2}=-\frac{-R\ddot{R}}{\dot{R}^2}$$

Equations (5)-(7) and (9) can be written in terms of H, σ and q as

$$H^2(2q-1)-\sigma^2 = 8\pi Gp - \Lambda \quad .....(14)$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \quad .....(15)$$

Overduin and Cooperstock (1998) define

$$\rho_c = \frac{3H^2}{8\pi G} \quad .....(16)$$

$$\rho_v = \frac{\Lambda}{8\pi G} \quad .....(17)$$

and $\Omega = \dfrac{\rho}{\rho_c} = \dfrac{8\pi G\rho}{3H^2}$ .....(18)

are respectively critical density, vacuum density and density parameter

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0 \quad .....(19)$$

From (15), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \dfrac{\sigma^2}{\theta^2} \le \dfrac{1}{3}$ and $0 \le \dfrac{8\pi G\rho}{\theta^2} \le \dfrac{1}{3}$ for $\Lambda \ge 0$

Thus, the presence of positive Λ puts restriction on the upper limit of anisotropy, where as a negative Λ contributes to the anisotropy.

From (14), and (15), we have

$$\frac{d\theta}{dt} = -12\pi Gp - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3\sigma^2}{2} = -12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative Λ, and for

positive Λ, universe will slows down the rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implying that σ decreases in an evolving universe and it is negligible for infinitely large value of R.

3. SOLUTION OF THE FIELD EQUATIONS -

The system of equations (3), (5)-(7), and (10), supply only five equations in six unknowns (A, B, ρ, p, G and Λ). One extra equation is needed to solve the system completely. Holy, F. et al (1997) considered Λ α a^{-3} whereas Λα a^{-m} (a is scale factor and m is constant) considered by Olson, T.S. et al. (1987), Pavon, D. (1991), Maia, M.D. et al. (1994), Silveria, V. et. al. (1994, 1997) , Torres, LF. B. et al. (1996).

Thus we take the decaying vacuum energy density

$$\Lambda = \frac{a}{R} \qquad .....(20)$$

where a is positive constants. Using eq. (12) and (20) in eq. (10), we get

$$G = \frac{a}{40\pi k}\frac{R^5}{} \qquad .....(21)$$

From eq (14), (15), (20) and (21) we get

$$\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 \quad - \frac{a}{R} = 0 \qquad ....(22)$$

Integrating (22) we get

$$R = \left[\frac{1}{2}\sqrt{\left(\frac{2a}{5}\right)}t + t_0\right]^2 \qquad .....(24)$$

where the integration constant to is related to the choice of origin of time. From (23) we obtain

$$R = \left[\frac{1}{2}\sqrt{\left(\frac{2a}{5}\right)}t + t_0\right]^2 \qquad .....(24)$$

By using (24) in (13) , the metric (1) assumes the form

$$ds^2 = -dt^2 + \left(\frac{1}{2}\sqrt{\frac{2a}{5}}\,t + t_0\right)^4 \times$$

$$\left[m_1^2 \ \exp\left\{\frac{8k_1}{3}\sqrt{\frac{5}{2a}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{2a}{5}}t+t_0\right)^{-5}\right\}dx^2 \right. \quad .....(25)$$

$$\left. +m_2^2 \ \exp\left\{\frac{-4k_1}{3}2\sqrt{\frac{5}{2a}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{2a}{5}}t+t_0\right)^{-5}\right\}(dy^2+dz^2)\right]$$

where m_1, m_2 are constants.

For the model (25), spatial volume V, matter density ρ, pressure p, gravitational constant G, cosmological constant Λ are given by

$$V=\left[\frac{1}{2}\sqrt{\left(\frac{2a}{5}\right)}t+t_0\right]^6 \quad ....(26)$$

$$\rho=p=\frac{k}{\left(\frac{1}{2}\sqrt{\frac{2a}{5}}t+t_0\right)^{12}} \quad .....(27)$$

$$G=\frac{a}{40\pi k}\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t+t_0\right]^{10} \quad ....(28)$$

$$\Lambda=\frac{a}{\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t+t_0\right]^2} \quad .....(29)$$

Expansion scalar θ and shear σ are given by

$$\theta=3\sqrt{\frac{2a}{5}}\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t+t_0\right]^{-1} \quad .....(30)$$

$$\sigma=\frac{k}{\sqrt{3}}\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t+to\right]^{-6} \quad ....(31)$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{1}{6} \quad .....(32)$$

The deceleration parameter q for the model is

$$q = -\frac{1}{2} \quad ..(33)$$

In the model, we observe that , the spatial volume V is zero at $t = \dfrac{-t_0}{\frac{1}{2}\sqrt{\frac{2a}{5}}} = t''$ and expansion scalar θ is infinite at t= t'' which shows that the universe starts evolving with zero volume and infinite rate of expansion at t= t''. Initially at t = t'' the energy density ρ, pressure p, Λ and shear scalar σ are infinite. As t increases the spatial volume increases but the expansion scalar decreases. Thus the expansion rate decreases as time increases. As t tends to ∞ the spatial volume V becomes infinitely large. As t increases all the parameters p, ρ, Λ, θ, ρ_c, ρ_v, decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that model approaches isotropy for large values of t. The gravitational constant G(t) is zero at t =t'' and as t increases, G increases and it becomes infinite large at late times.

Further, we observe that $\Lambda \alpha \frac{1}{t^2}$ which follows from the model of Kalligas et al. (1996); Berman (1990); Bertolami (1986a, b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

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SOME FEATURES OF CHARGE CARRIER TRANSFER IN GRANULAR SEMICONDUCTORS - EXPERIMENT

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A B S T R A C T

Abstract: In the present paper, in accordance with the structural model and location of granules developed earlier, the mechanisms of carrier charge transfer in granular semiconductors have been experimentally investigated. Two types of location of the granule are found in thermoelements based on granular silicon. It has been experimentally established that for the first type there is an increase the specific resistance, while for the second type, the specific resistance decreases with increasing temperature. To the first type the charge transfer from the first granule to the second will occur through the regions between the granular boundaries, and the second type the process of charge transfer occurs mainly along the regions between the granular boundaries that a formed when parallel to each other. In was found a full accordance mechanism of occurrence of carrier charge transfer in granular semiconductors have been and experiment.

Keywords: structura, specific resistance, semiconductor, potential barrier, move levels, localized traps, granula, conductivity of the traps, charge transfer.

1. Introduction

Within the framework of representations developed in the first part of our work [1], we suggested simple answer to the issue what the mechanism of charge transfer in granular semiconductors is. It was believed that the processes of charge transfer in granular semiconductors depend on the size, structure and location of the granules. That is, the location of the granules can be in two forms, e.g., in series or in parallel next to each other. Accordingly, the transfer process of charge carriers in them is fundamentally different. In the first variant, i.e., if the granules are arranged in series, the transfer of the charge from the first granule to the second granule will occur through the two contacting regions. In this case, the increase in trapped charge on localized traps in the two contacting regions leads to an increase in the height of the potential barrier, and this simultaneously leads to an increase in the resistivity. To the second variant, if, the granules are arranged parallel to each other, the contacting area, also forms parallel between the granules as in the first variant, local energy levels are formed in it. However, in this case, the charge carriers do not move from one granule to the other, they are trapped in traps (E_i levels), that are parallel to the formed contacting regions, and move to $E_i$levels. Wherein, there is an increase in the total conductivity of the traps (Y_{ss}). In this case, the overall specific conductivity is determined by the total specific conductivities. In this connection, in the present work, we discuss the experimental results of the charge transfer process in granular silicon, and the proposed mechanism is specified.

II. RESEARCH METHOD

As mentioned in the previous article [1], there are a number of methods for obtaining granular semiconductor materials, of which the powder technology method is a promising method for producing polycrystalline semiconductor materials for solar cells or integrated circuits, as well as for thermoelements. Silicon granules were obtained in the framework of powder technology, which are given in work [2 ÷ 5], and have been used by us for obtaining a polycrystalline silicon plate. As a raw material of the technology, single-crystal silicon substrates were used, where solar cells with a coefficient of about 15% are used on their basis. The starting samples are ground in a ball mill, wherein the powder can be obtained with a granules size of ten to a thousand nanometers controlling the grinding time. In order to research structure of the granular silicon electronic microscope was used and to assess the transfer of charge carriers the specific resistance (ρ) of the granules was measured by two-probe method in a temperature range of 20 to 300 °C [4]. The measurements were conducted in semi-automatic mode directly with the granules heated, as in a temperature rise from 20 to 300 ° C, and at the stage of its decrease.

III. EXPERIMENT

As mentioned above and in the previous article [1], the granules can be arranged in two variants, in series or parallel to each other. First, let's see the first variant.

Variant 1. Sequential location of granules. (Fig. 2). Usually such structures are observed larger sizes, for example, in granules with sizes of 100 ÷ 1000 micrometers. Figure 1 illustrates the temperature dependence of the specific resistance (ρ) of granular silicon, (1 -1000 μm, 2 - 300 μm and 3 - 100 μm). From Fig. 1, it can be seen that the specific resistancein all cases increases with temperature. With decreasing granules size (1→2→3), the growth in specific resistance is observed more strongly than in others.

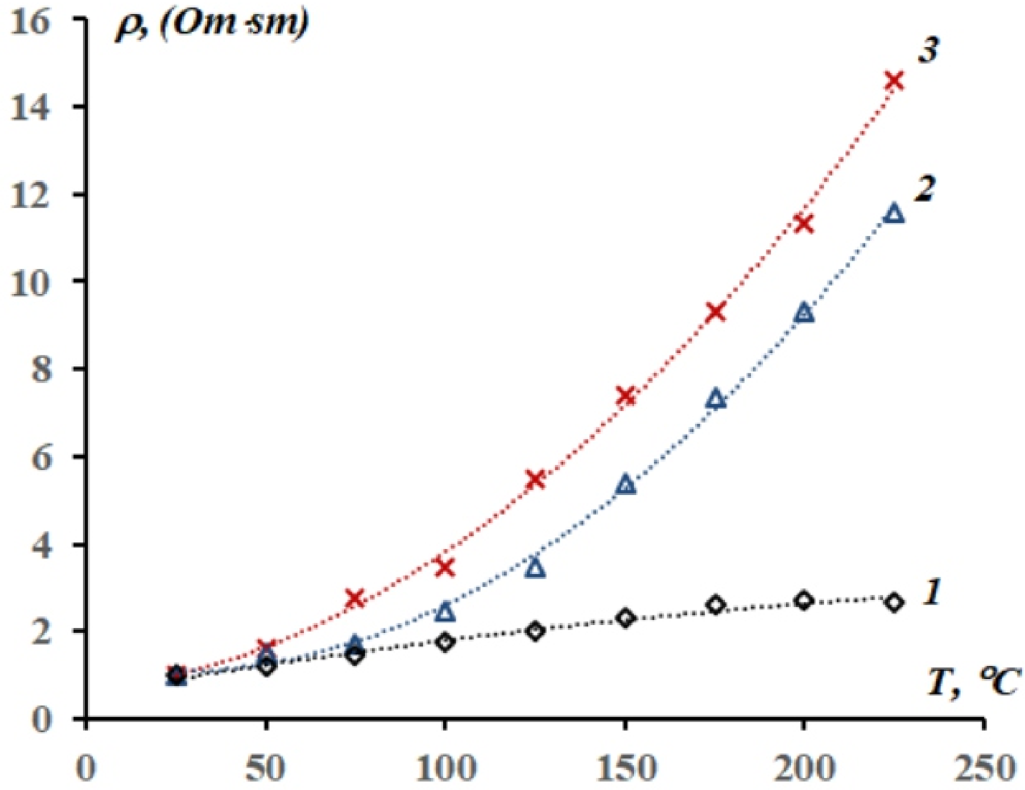

Fig.1. Dependence of the specific resistance on temperature

From the proposed mechanism for the emergence of carrier-charge transfer in granular semiconductors, in the first part of our work [1] it is known that the specific resistance is described by the expression:

$$\rho = \frac{k}{q\langle a\rangle A^* T}\exp\left({}^{q\varphi}\!/_{kT}\right) \qquad (1)$$

where <*a*> is the grain size, φ is the potential barrier height on the grain boundary.The potential barrier height

$$\varphi = \frac{Q_i^2}{8\varepsilon\varepsilon_0 q N_G} \qquad (2)$$

Here, Q_i is the boundary derivative charge in the grain boundaries, N_G - is the concentration of an electrically active dopants, ε_0 and ε – dielectric constant.

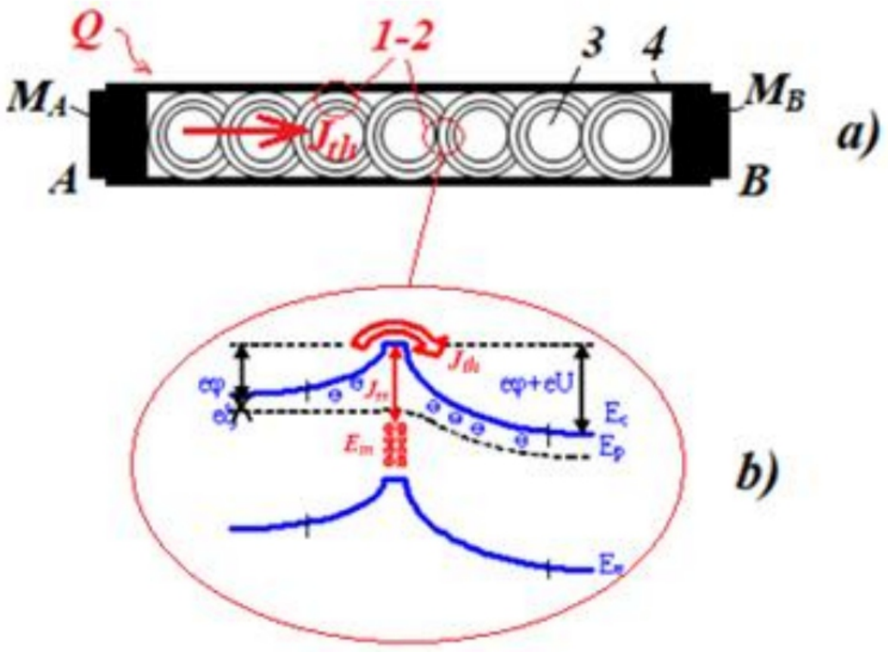

Fig.2. Simplified scheme of location of the granule (a), zone diagram (b). (1, 2, 3) - silicon granules, 4 - ceramic tube, M_A and M_V ohmic contacts respectively, in the A and B regions

It is seen from (1) that an increase in the trapped charge Q_i on localized traps in the boundaries between the granules, that is, in the two contacting regions of the granule, leads to an increase in φ, and this simultaneously leads to an increase in ρ (Fig. 2a). An equivalent electrical scheme of such a structure is presented on fig.2. It can be seen that the equivalent electrical scheme consists of the series-connected resistances of the granule and the two contacting regions of the granule. In this case, the total specific resistance is determined by the total specific resistance's. In addition, it can be seen from (1) that the specific resistance is related to the granule size $<a>$, that is, a decrease in the granule size $<a>$ leads to an increase in ρ. From Fig. 1, it is seen that with a decrease in the size of the granule (1→2→3), the growth of resistivity is observed more strongly than in others. This indicates that an increase in the trapped charge Q_i on localized traps in the boundaries between the granules, in smaller sizes is observed more strongly.

Variant 2. Arrangements of granule next to each other (in Fig. 6). Usually such structures are observed in smaller sizes, for example, in granules with dimensions of 400 ÷ 700 micrometers. Figure 3 illustrates the temperature dependence of the specific resistance (ρ)of granular silicon, it is very different from the first. From Fig. 3, it is seen that the resistivity decreases exponentially, and the dark current increases with temperature. (Fig. 4). It also shows a decrease in voltage and growth (Figure 5). To explain the results we use the model of thermionic emission [4÷7] with additions concerning the account of the currents arising in the process of capture and emission of charge carriers in traps, which we formulated in

[1], and also by a simplified sample scheme (Figure 6a) and the zone diagram between the charged granules (Fig. 6b).

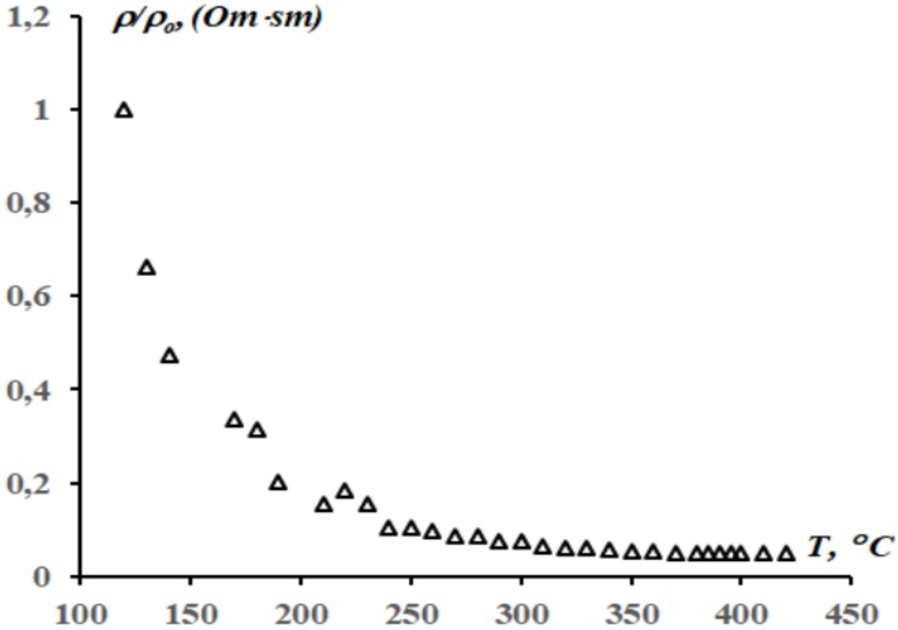

Fig. 3. Dependence of the specific resistance on temperature

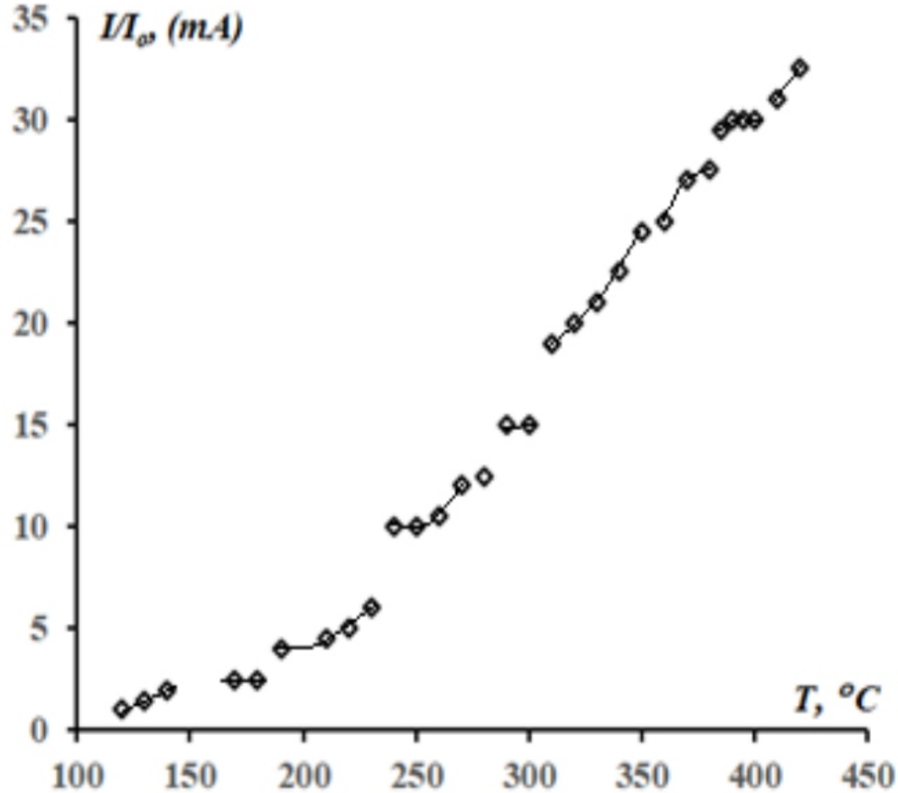

Fig. 4. Dependence of dark current on temperature

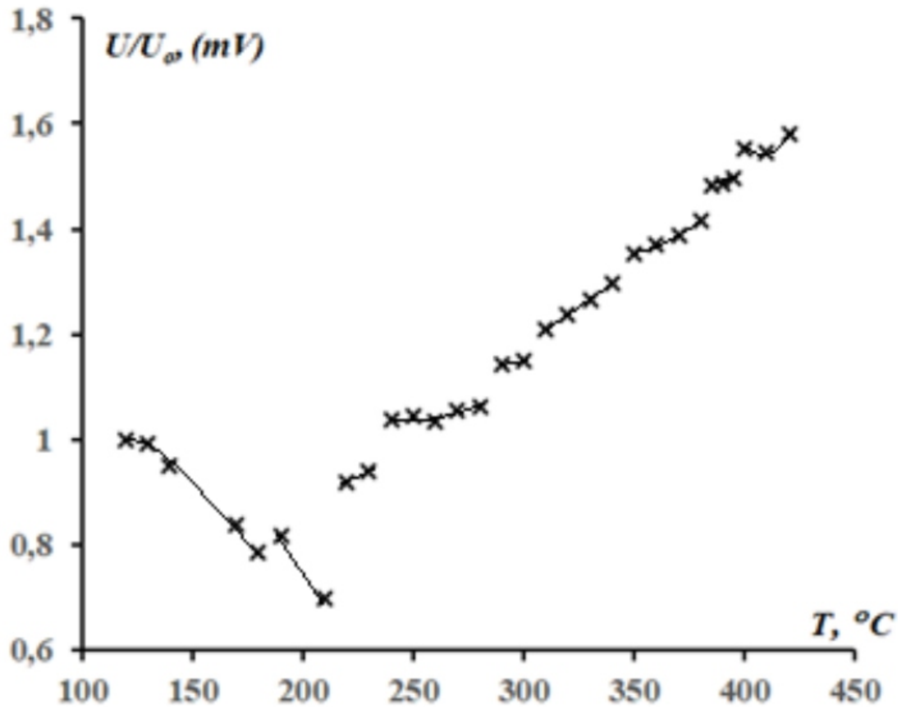

Fig. 5. Dependence of voltage on temperature

It is known that for semiconductors the decrease in the resistivity is associated with the onset of intrinsic conductivity with increasing temperature. And also, this increases the concentration of charge carriers and this in turn leads to an increase in current. The result of the research confirms this mechanism. However, according to the location of the granules, as described in the first part of our work [1] (Fig. 6), the charge transfer process must occur in two adjacent regions.

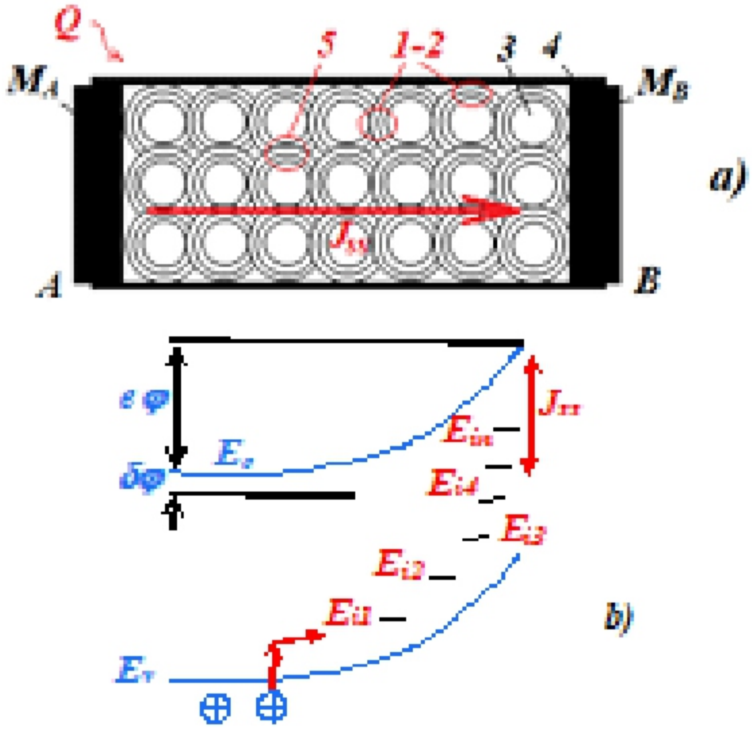

Fig.6. Simplified scheme of location of the granule (a), zone diagram (b). (1, 2, 3) - silicon granules, 4 - ceramic tube, MA and MB contacts, respectively, in the A and B regions

From Fig. 6b, it follows that charge carriers are captured by states (E_i) at the boundary region. Emerged charge is compensated by ionized charges located in the space charge region. During the capture and emission of charge carriers on traps observed J_{ss} currents arise and they move along E_i levels located in region 5 (Figure 6). The current J_{ss} is:

$$J_{SS} = Y_{SS}\delta\varphi \tag{3}$$

where Yss is the characteristic total conductivity of the traps, which depends both on their capture cross section and on the energy distribution, as well as on the position in space, that is, the location in region 5 (Fig. 6), $\delta\varphi$ is the change in the height of the potential barrier. The current Jss is identically equal to the derivative in time with respect to charge bound on the boundary surface. In region 5 (Fig. 6) the following situation takes place [4, 6, 8]: to the process of hole capture and emission on the boundary surface, the width of the spatial charge region should vary in order for total electroneutrality to be kept. This, in turn, influences the whole zone diagram (Fig. 6b), i.e., as a change in the barrier height δφ, so and on Yss. This means that the current Jss and the change in the barrier height $\delta\varphi$ are interrelated, and the vibrational properties of this interrelation are determined by the properties of the traps. Naturally, in the process of temperature change, both capture and emission of charge carriers with the participation of traps are observed. When the number of charge carrier captures is greater than their emission, the motion of the charge along the

levels of the traps along the boundary of the two contacting granules (in region 5, Fig. 6a) is accompanied by a growth in the total conductivity of the traps. If the emission prevails over the capture, then the conductivity of the traps also increases, despite the fact that the charge moves in the opposite direction.

In this work [4, 6, 8], we studied the manifestations of the energy states conditioned by traps, for example, with $E_{i1} \sim 0{,}15\ eV$, $E_{i2} \sim 0{,}17\ eV$, $E_{i3} \sim 0{,}36\ eV$, $E_{i4} \sim 0{,}3\ eV$ which are manifested at temperatures s T~70÷420 °C. Current changes for example, growth at T≤150 °C, T~180÷230 °C, T~240÷280 °C, T~290÷300 °C, T~310÷380 °C, T≥400 °C, decrease at T~150÷170 °C, sharp growth at T~180 °C, T~240 °C, T~290 °C, T~310 °C, and also voltage changes at these temperatures correspond to manifestations due to the energy states $E_{i1}, E_{i2}, E_{i3}, E_{i4}$. The outgoing carriers from the valence band Ev are captured in E_{i1} and they successively move through levels that exhibit states E_{i2}, E_{i3}, E_{i4} with temperature changes (Fig. 6b). This process occurs before these levels are filled. During the transition of charges over levels, the current Jss and the conductivity of the Yss traps [4÷7] arise, and this simultaneously leads to a decrease in the resistivity. Thus, the comparison of the qualitative theory given in [1] with the experiments discussed here shows almost complete coincidence, the simplicity of the mechanism and its correspondence to other phenomena in granular semiconductors indicate its viability.

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